



Mixed Integer Linear Programming Software Development Using Visual Basic Programming Language

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Abstract

This paper focused on developing software for the mixed integer linear programming problem which will enhance effective policy formulation. Visual basic programming language was used for the implementation of the mixed integer linear programming model because of the nature of the problems it solves and time saves benefits. The application was tested and debugged. The authentication was ensured by using benchmark examples of mixed integer linear programming problems found on standard books, the model professionally simulated with the result printed out successfully.

Keywords: Mixed integer linear programming, linear programming, Object-oriented programming, visual basic, Computer aided design CAD.

Introduction

Finding the best result for a given problem under certain conditions in real-world, different fields of science, engineering and optimization problems are subject to different types of objective functions and constraints with different types of variables (Mohamed A. , 2017), (Mohamed & Sabry, 2012). So, most of these problems can be expressed as mixed integer non-linear programming problems (MINLP) that involve continuous integer decision variables. These problems are recognized as a class of NP-complete problems due to their combinatorial nature and are considered

difficult problems (Costa & Oliveira, 2001), (Lin, Hwang, & Wang, 2004). This class of optimization problems frequently arises in real-world many application fields such as mechanical design (Sandgren, 1990), synthesis of chemical process flow sheets and design of materials (Dua & Pistikopoulos, 1998), scheduling (Catalaño, Pousinho, & Mendes, 2010), network design (Garroppo, Giordano, Nencioni, & Scutella, 2013), feature selection (Maldonado, Pérez, Weber, & Labbe, 2014), vehicle routing (Cetinkaya, Karaoglan, & Gökcen, 2013), strategic planning

(Liu, Whitaker, Pistikopoulos, & Li, 2011), data classification (Xu & Papageorgiou, 2009), drones fly Sidekick for military and civilian applications, including logistics delivery (Kuroswiski, Pires, Passaro, Guimarães, & Senne, 2023) and many more (Grossmann & Sahinidis, 2002). mathematical formulations in Mixed Integer Linear Programming and proposes a hybrid Genetic Algorithm (HGenFS) for optimizing a variation of the Traveling Salesman Problem (TSP) called Flying Sidekick TSP (FSTSP), in which truck and drone cooperate (Kuroswiski, Pires, Passaro, Guimarães, & Senne, 2023). Mixed-integer optimization problems can be computationally challenging (Chamanbaz & Bouffanais, 2023), Solving mixed integer linear programming (MILP) problems is a difficult task due to the parallel use of both integer and non-integer values (Abaffy & Fodor, 2013). In the fields of science and engineering, researchers have a great interest in solving optimization problems consisting of real and discrete variables, which arise from a variety of real-world situations and applications (Sun & Gao, 2019). (Awwalu, Abdullahi, & Hussaini, 2023) Use mixed-integer programming to solve the reliability problem of water production, reduce the design and operational costs of water distributions. This study aims at providing a visual display of the solution, speedily interpret the optimal solution by enhancing computational complexity, reducing laborious tasks encountered in solving industrial model mixed integer linear programming problems, providing very accurate and proficient results.

Materials and Method

Preliminaries of Software Model Development

To develop this program, we need the following definitions of the basic arithmetic operators as presented by (Sharma, 2010).

Consider the following mixed integer programming problem

$$\text{Maximize } Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

Subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\dots \dots \dots \dots \dots \dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

And

$$x_j \text{ and integer; } j = 1, 2, \dots, k \text{ (} k < n \text{)}.$$

If the basic variable x_r is restricted to be integer and has a larger fractional value in the basic variables restricted to take integer values. The rewrite r th constraint is presented as.

$$x_{Br} = x_r + \sum_{j \neq r} a_{rj} x_j \tag{1}$$

Where x_j represents all the non-basic variables in the r th row except variable x_r , and x_{Br} is the non-integer value of variable x_r .

Decompose coefficients of x_j , x_r , variables, and x_{Br} into integer and non-negative fractional parts as shown below.

$$x_{Br} = [x_{Br}] + f_r$$

And

$R_+ = \{j : a_{rj} \geq 0\}$, Set of subscripts j (columns in the simplex table) for which $a_{rj} \geq 0$

$R_- = \{j : a_{rj} < 0\}$, Set of subscripts j (columns in the simplex table) for which $a_{rj} < 0$

Then, we can rewrite Equation 1, as

$$[x_{Br}] + f_r = (1 + 0)x_r + \sum_{j \in R_+} a_{rj} x_j + \sum_{j \in R_-} a_{rj} x_j \quad 2$$

Rearrange equation 2, to put integer coefficients on the right-hand side. This gives

$$\sum_{j \in R_+} a_{rj} x_j + \sum_{j \in R_-} a_{rj} x_j = f_r + \{[x_{Br}] - x_r\} = f_r + 1 \quad 3$$

Where f_r is a strictly positive fractional number (i.e. $0 < f_r < 1$), and I is an integer value.

Since the terms in the bracket on the right-hand side of equation 3, are integers, left-hand side in equation 3, is either positive or negative according as $f_r + 1$ is positive or negative.

Case 1, let $f_r + 1$ be positive. Then this must be $f_r, 1 + f_r, 2 + f_r, \dots$, we shall then have

$$\sum_{j \in R_+} a_{rj} x_j + \sum_{j \in R_-} a_{rj} x_j \geq f_r \quad 4$$

Since $a_{rj} \in R_-$, are non-integer and $x_j \geq 0$,

$$\sum_{j \in R_+} a_{rj} x_j \geq \sum_{j \in R_+} a_{rj} x_j + \sum_{j \in R_-} a_{rj} x_j$$

And hence $\sum_{j \in R_+} a_{rj} x_j \geq f_r$

Case 2, let $f_r + 1$ be negative, then it must be $f_r, -1 + f_r, -2 + f_r, \dots$, and we shall have

$$\sum_{j \in R_-} a_{rj} x_j \leq \sum_{j \in R_+} a_{rj} x_j + \sum_{j \in R_-} a_{rj} x_j \leq -1 + f_r \quad 5$$

Multiplying both sides of equation 5, by the negative number $(\frac{f_r}{f_r-1})$, we get

$$\left(\frac{f_r}{f_r-1}\right) \sum_{j \in R_-} a_{rj} x_j \geq f_r \quad 6$$

Either of inequalities 4 and 6, holds since in both cases, the left-hand side is non-negative and one of these is greater than or equal to f_r . Thus, any feasible solution to MIP must satisfy the following.

$$\sum_{j \in R_+} a_{rj} x_j + \left(\frac{f_r}{f_r-1}\right) \sum_{j \in R_-} a_{rj} x_j \geq f_r \quad 7$$

Inequality 7, is not satisfied by the optimal solution of the LP problem without integer requirement. This is because by putting $x_j = 0$ for all j , the left-hand side becomes zero, and the right-hand side becomes positive. Thus, inequality 7, defines the cut.

Adding a non-negative slacks variable we can rewrite equation 7, as

$$S_g = -f_r + \sum_{j \in R_+} a_{rj} x_j + \left(\frac{f_r}{f_r-1}\right) \sum_{j \in R_-} a_{rj} x_j \quad 8$$

Equation 8, represents the required Gomory's cut

For generating the cut equation 8, it was assumed that the basic variable x_r should take an integer value, but if one or more $x_j, j \in R$ are restricted to be integers to improve a cut as shown

in eqn 7, to a better cut, i.e. the coefficients $a_{rj}, j \in R +$ and $a_{rj} \left\{ \left(\frac{f_r}{f_r-1}\right) \right\}, j \in R$. If is desired

to be as small as possible, we can proceed as follows. The value of the coefficients of x_j can be increased or decreased by an integral amount in equation 3, to get the term with the smallest coefficients in equation 7. To reduce the feasible region as much as possible through cutting planes, the coefficients of integer variable x_j must be as small as possible. The smallest positive coefficient for x_j in equation 3 is

$$\{f_{rj}; \frac{f_r}{1 - f_r} (1 - f_{rj})\}$$

The smaller among the two coefficients will be considered to cut equation 8, and penetrate deeper into the original feasible region. A cut is said to be deep if the intercepts of the hyperplane represented by a cut with the x-axis are larger obviously.

$$f_r \leq \frac{f_r}{1-f_r}(1-f_{rj}), f_{rj} \leq f_r \text{ and } f_r > \frac{f_r}{1-f_r}(1-f_{rj}), f_{rj} > f_r$$

Thus the new cut can be expressed as,

$$s_g = f_r + \sum_{j \in R} f_{rj}^* x_j \quad 9$$

Where

$$f_{rj}^* = \begin{cases} a_{rj}, & a_{rj} \geq 0 \text{ and } x_j \text{ integer} \\ \left(\frac{f_r}{f_r-1}\right) a_{rj}, & a_{rj} < x_j \text{ and } x_j \text{ integer} \\ f_{rj}, & f_{rj} \leq f_j \text{ and } x_j \text{ integer} \\ \left(\frac{f_r}{f_r-1}\right) (1-f_{rj}), & f_{rj} > f_j \text{ and } x_j \text{ integer} \end{cases} \quad 10$$

Program Description

The Mixed integer linear programming problems were created using the basic equation 1 to 10 modeled with visual basic.net object-oriented programming language (Deitel & Deitel, 2010). The flowchart, program algorithm and the method of input, analysis, and outputting of date is presented.

Input Phase

(Deitel & Deitel, 2010) Information is obtained using window form, textbox, and command prompt for collecting input data.

Analysis

Codes were assembled in correct syntax to excellently analyze the input dates to supply accurate result of the objective function (Z), the basic variables (X_i) as output and the result displayed depend on the computer speed.

Output Phase

This shipping section takes information that the computer has processed and places it on various output devices to make it available for use outside the computer (Deitel & Deitel, 2010).

Algorithm

The pseudocode used in the development of this software was presented as follows;

Start

Mixed integer linear programming

Input known parameters of A and C of x_j using the appropriate equation 1 to 10.

Input known parameters (constant) B using the respective equation 1 to 10

Compute desired parameters Z and x_j using equations 1 to 10

If x_j equals integer value then
 Z is the optimal solution
 End if
 Else if $x_j =$ non integer value then
 Select x_j with the highest fraction using respective equations 1 to 10
 Generate cutting plane using equations 1 to 10
 Add a cutting plane to the bottom of the optimal solution
 Compute desired parameters Z and x_j using respective equations 1 to 10
 If $x_j =$ integer value then
 End if
 End if
 End

Flow chart

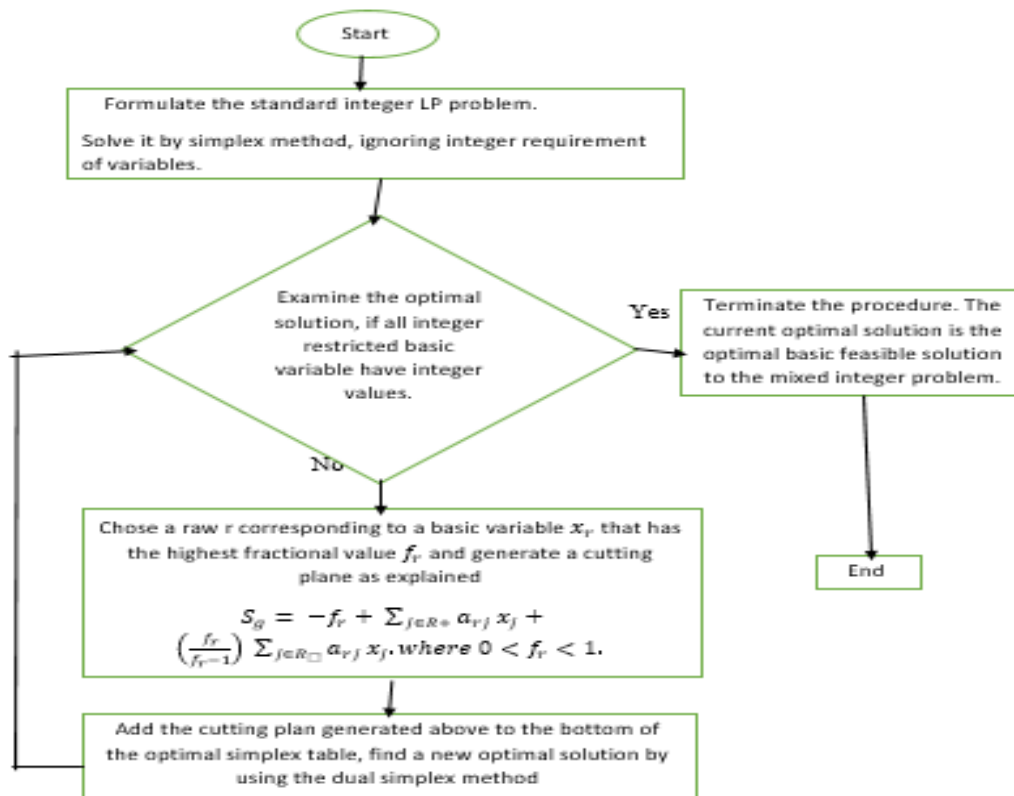


Figure 1, shows the flow chart above.

Examples to Illustrate the Use of the Software

Example 1

Solved the following mixed integer programming problem.

$$Z = -3x_1 + 3x_2 + 3x_3$$

Subject to the following constraints

$$\begin{aligned}
 -x_1 + 2x_2 + x_3 &\leq 4 \\
 2x_2 - 1.5x_3 &\leq 1 \\
 x_1 - 3x_2 + 2x_3 &\leq 3 \\
 \text{and } x_1, x_2 &\geq 0, x_3 \text{ non - integer.}
 \end{aligned}$$

The software solution to example 1 is presented in Table 1 below



Example 2.

Solved the following mixed integer programming problem.

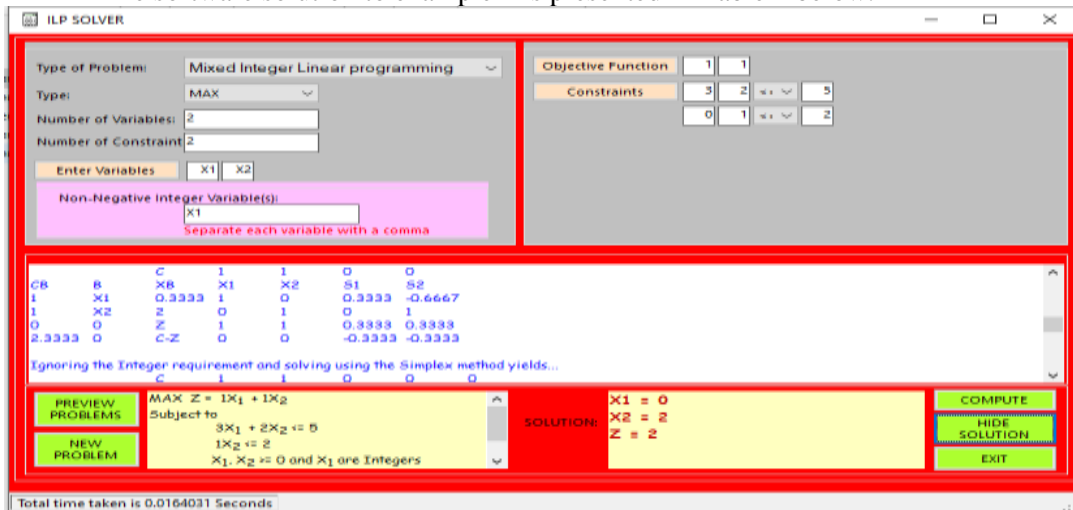
$$Z = x_1 + x_2$$

Subject to the following constraints

$$\begin{aligned}
 3x_1 + 2x_2 &\leq 5 \\
 x_2 &\leq 2
 \end{aligned}$$

and $x_1, x_2 \geq 0, x_1$ non - integer.

The software solution to example 2 is presented in Table 2 below.



Conclusion

The mixed integer linear programming software was developed to ease the manual computational complication in solving industrial base model problems and as an aid to

supervisors by providing the best solutions to a diversity of mixed integer linear programming problems with the purpose of optimization of production, maximizing profit and reduced time lost from the manual computations.

Reference

- Abaffy, J., & Fodor, S. (2013). Solving Integer and Mixed Integer Linear Problems with ABS Method. *Acta Polytechnica Hungarica*, 10(7), 81-98.
- Awwalu, H., Abdullahi, N., & Hussaini, M. (2023). *A conceptual model of mixed integer linear programming water distribution system* (Vol. 4). Mathematics in Applied Sciences and Engineering.
- Catalão, J., Pousinho, H., & Mendes, V. (2010). Mixed-integer nonlinear approach for the optimal scheduling of a head-dependent hydro chain. *Electr Power Syst Res*, 80(8), 935–942.
- Cetinkaya, C., Karaoglan, I., & Gökcen, H. (2013). Two-stage vehicle routing problem with arc time windows: a mixed integer programming formulation and a heuristic approach. *Eur J Oper Res*, 539–550.
- Chamanbaz, M., & Bouffanais, R. (2023). A Sequential Deep Learning Algorithm for Sampled Mixed-integer Optimisation Problems. *Journal of Information Sciences*.
- Costa, L., & Oliveira, P. (2001). Evolutionary algorithms approach to the solution of mixed non-linear programming. *Comput Chem Eng.*, 257–266.
- Deitel, P., & Deitel, H. (2010). *Microsoft Visual Basic 2010, How to programme*. New Jersey 07458: Microsoft published by pearson Education, Inc Upper Saddle River.
- Dua, V., & Pistikopoulos, E. (1998). Optimization techniques for process synthesis and material design under uncertainty. *Chem Eng Res Des* 76(3):408–416, 408–416.
- Garropo, R., Giordano, S., Nencioni, G., & Scutella, M. (2013). Mixed integer non-linear programming models for green network design. *Comput Oper Res*, 273–281.
- Grossmann, I., & S. N. (2002). Special issue on mixed-integer programming and its application to engineering. (N. Kluwer Academic Publishers, Ed.) *Part I, Optim. Eng., Kluwer Academic Publishers, Netherlands.*, 3(4).
- Jayalakshmi, P. P. (2010). A New Method for Solving Integer Linear Programming Problems with Fuzzy Variables. *Applied Mathematical Sciences*, 4(20), 997 - 1004.
- Kuroswiski, A., Pires, H., Passaro, A., Guimarães, L., & Senne, E. (2023). Hybrid Genetic Algorithm and Mixed Integer Linear Programming for Flying Sidekick TSP. *inf and sc*. Retrieved from www.researchgate.net/publication/370338954
- Lin, Y., Hwang, K., & Wang, F. (2004). A mixed-coding scheme of evolutionary algorithms to solve mixed-integer nonlinear programming problems. *Comput Math Appl*, 1295–1307.
- Liu, P., Whitaker, A., Pistikopoulos, E., & Li, Z. (2011). A mixed-integer programming approach to strategic planning of chemical centres: a case study in the UK. *Comput Chem Eng*, 1359–1373.
- Maldonado, S., Pérez, J., Weber, R., & Labbe, M. (2014). Feature selection for support vector machines via mixed integer linear programming. *Inf Sci*, 163–175.
- Mohamed, A. (2017). An efficient modified differential evolution algorithm for solving constrained non-linear integer and mixed-integer global optimization problems. *Int. J. Mach. Learn. & Cyber.*, 989-1007.
- Mohamed, H., & Sabry, A. (2012). Constrained optimization based on modified differential. *Information Sciences*, 171-208.
- Sandgren, E. (1990). Nonlinear integer and discrete programming in mechanical design. *ASME Y. Mech Des*, 223–229.

- Sharma, j. k. (2010). *Operations Research theory and applications*. (4, Ed.) Rajiv Beris for Macmillan publishers India Ltd.
- Sun, Y., & Gao, Y. (2019). *An Efficient Modified Particle Swarm Optimization Algorithm for Solving Mixed-Integer Nonlinear Programming Problems* (Vol. 12). International Journal of Computational Intelligence Systems. doi: <https://doi.org/10.2991/ijcis.d.190402.001>
- Xu, G., & Papageorgiou, L. (2009). A mixed integer optimisation model for data classification. *Comput Ind Eng*, 1205–1215.