



Comparative Analysis of Triplets Surface Determined with DGPS and Automatic Level

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Abstract

Dual Frequency Global Positioning System (DGPS) has emerged as a successful technology in providing precise positions of points on the surface of the earth over the reference ellipsoid with sub-metre level of accuracy. DGPS is one of the most frequently used positioning methods in geodesy. The end products of surveying with this receiver gives geodetic latitude (ϕ), geodetic longitude (λ) and ellipsoidal height (h) which were obtained with reference to the ellipsoid. This research involved the determination of Geoidal undulation for the production of Geoidal map. The objectives include the determination of orthometric, ellipsoidal and geoidal heights, to use orthometric and ellipsoidal height for the purpose of aspect and slope map comparison. DGPS receiver and precise Level instruments were used to obtain ellipsoidal and orthometric heights of the study area respectively. Geoidal heights were derived from the differences between ellipsoidal heights and orthometric heights. The adjusted orthometric heights obtained from precise Level and the ellipsoidal heights which are part of the geodetic/Universal Transverse Mercator (UTM) coordinates obtained from DGPS were post processed using spectrum survey office Software. The Geoidal map, aspect map and slope map were created using ArcGIS 10.2.1 Software version. The Microsoft Office Excel was used to deduce the ellipsoidal heights, orthometric heights and geoidal undulations for the production of Geoidal Map of the study Area. The statistical analysis of the result met the precision of second order geodetic control network and levelling specifications. The mean value of the geoidal heights determined is 43.650 metre which could be used as the geoid within the study area.

KEYWORDS: *Global Positioning System, Orthometric, Ellipsoidal and Geoidal heights*

Introduction

BACKGROUND TO THE STUDY

One of the basic goals of geodesy is the determination of the geoid which is the equipotential surface of the earth gravity field that coincides on the average with the mean sea level. The geoid is the surface which coincides with that surface to which the oceans would conform over the entire earth, if free to adjust to the combined effects of the earth's mass attraction (gravitation) and the centrifugal force of the Earth's rotation. Specifically, it is an equipotential surface, meaning that it is a surface on which the gravitational potential energy has the same value everywhere with respect to gravity. The geoid surface is irregular, but considerably smoother than earth's physical surface. Sea level, if undisturbed by tides, currents and weather, would assume a surface equal to the geoid (Featherstone 2000).

Geodesy is concerned with the relative positioning of points and the gravity field of the earth. For geoidal mapping and three dimension (3D) surface modelling, a well-defined coordinate system is needed on which measurements are tied to a set of reference points called a geodetic datum (geoid or ellipsoid). A control survey is a means of establishing precise positions of geodetic monuments. There are two types of survey controls. These include horizontal and vertical controls. Horizontal controls are defined with respect to an ellipsoid of revolution whilst vertical

controls are defined with reference to a local geoid. Horizontal and vertical terrestrial geodetic control networks are important and valuable for the accurate positioning of construction and engineering projects, they serve as points of reference for correct positioning. Controls are established by classical and modern methods. The classical methods are traversing, triangulation and trilateration while the modern methods include the use of satellite techniques such as the Global Positioning System (GPS), and satellite altimeters. Satellite techniques could be used to establish and densify 3D networks more rapidly, with greater accuracy and less difficulty than terrestrial techniques (Poku and Gunter, 2006).

Geoid comes from the word "geo" which literally means earth-shaped. Geoid is an empirical approximation of the figure of the earth (minus topographic relief). It is defined as the equipotential surface of the earth's gravity field which best fits, in the least square sense; the mean sea level. On the ocean, the geoid is on average at the same level as mean sea level, the surface obtained by removing from the instantaneous sea surface all periodic and quasi-periodic variations (tidal phenomena, air pressure, littoral seas, eddies and continual shifting of ocean currents).

Ellipsoids are reference surfaces usually determined on the physical surface of the earth. The difference in height between the

physical surface and the ellipsoid is regarded as the ellipsoidal height and the difference in height between the physical surface and the geoid is called the orthometric height. The difference in height between the geoid and the ellipsoid is regarded as the geoidal height or geoidal undulation. The height anomaly is derived from the difference between the quasi-geoid and the ellipsoid or the physical surface of the earth and the telluroid. The height anomaly is a quantity similar to the geoid height, however is located on the level of the topography not sea level. The surface formed by points which are above the reference ellipsoid (and thus a distance below the topography), is called the telluroid. The surface formed by points which are above the reference ellipsoid which coincide with the geoid at sea level, if free to adjust to the combined effects of the earth's mass attraction and the centrifugal force of the Earth's rotation is called the quasi-geoid. It lacks any physical meaning; it is not an equipotential surface, although at sea it coincides with the geoid. Normal heights are very operational. They are always used together with so-called "quasi-geoid" heights (more correctly: height anomalies). orthometric heights (more precisely: Helmert heights) on the other hand are always used together with geoid heights. Presently, the most accurate positioning technology is the Global Positioning System (Ayhan et al, 2009). GPS gives accurately the 3D position of points (ellipsoidal latitude, longitude, and height) and can measure under favourable weather conditions. In addition, it can measure when placed on any platform (static or dynamic). One major advantage of GPS technology over the traditional methods is that inter-visibility is not a requirement. In addition to providing highly accurate data, it is easy to use, portable, less labour intensive, and its surveys are relatively less costly. The coordinates of the GPS are referenced to the World Geodetic System 1984 (WGS 84), a global ellipsoid having its origin closed to the earth centre of mass, which forms the origin of its coordinate system. GPS applications range from cadastral surveys to monitoring sea level rise; from navigation and mapping to the use of remote sensing for resource management; from mineral exploration to assessment of potential flooding areas; from the construction and precise positioning of dams and pipelines to the interpretation of seismic disturbances. The height reference system is also implicated in many legal documents related to land management and safety such as easement, flood control, and boundary demarcation (Ayhan et al, 2009). Therefore, based on these information and the needs to support modern height system, the implementation of Accurate Height System (AMS) in Nigeria particularly in Bauchi metropolis has to be realized.

There are a lot of geodetic methods for determining of height or heights differences. These methods are classified as geometric levelling, trigonometric levelling, and GPS/Levelling. Generally, geoid undulation is required for many geodetic and surveying applications. The most notable application is for converting GPS-derived ellipsoidal height to orthometric height for engineering. Classical methods of heights determination is the techniques usually adopted in the study area; hence there is high demand for digital dataset for the production of Geoidal map of the study area. There is a growing use of GPS surveys in

Nigeria due to its numerous advantages over classical methods. If GPS data is properly processed and used, GPS can be an effective tool to promote national development as the data can be used for planning of communities, exploration and exploitation of natural resources, correct positioning of engineering and construction works, scientific investigation, enhancement of agricultural productivity and provision of services among other applications of which the study area is not an exception.

Several methods have been developed for geoid determination, which are classified as geometric approach and gravimetric approach. In geometric approach local geoid can be determined from a relatively small and flat area by combination of GPS derived heights and leveled heights (In this thesis, orthometric heights will be determined using precise levelling equipment). From the GPS derived heights and leveled heights at some points of interest, the geoid heights at these points can be calculated. The geoid heights at any other points can be interpolated analytically or graphically based on the known geoid heights. The elevation of a station is a vertical distance above or below level surface. The approximate surface is the MSL which fit to the geoid surface. Another surface is the surface of the ellipsoid. The difference in elevation between two stations is the value of the vertical distance between the two different level surfaces which this elevation assigned to it. Global Positioning System (GPS) and precise/geodetic levelling are the modern technologies adopted in determining the heights above the ellipsoid and geoid respectively.

Geoidal height or Geoid undulation is considered as the separation of the reference ellipsoid with the geoid surface measured along the ellipsoidal normal (Erol, 2004). Geoid is also said to be an equipotential surface of the earth's gravitation field which coincides with the sea surface, in the absence of disturbing factors like tsunamis, ocean currents, salinities, wind, and so on (Vanicek and Krakiwsky, 1986). It is used to determine the shape of the earth in Geodesy. It also plays an important role in many other geosciences. It is a fundamental reference surface for surveying and mapping from which the natural elevation of a point is measured. Geoid is therefore, an equipotential surface that manifests the true distribution of masses in the interior of the earth; and is used by Geophysicist to obtain gravity anomalies in order to learn about the density variations of the earth interior (Anthony, 2011).

Besides this geometrical aspect of the geoid, it also related to the gravitational field of the Earth. It is actually possible to calculate the gravity accelerations everywhere outside the earth through analytical continuation if we know the gravity at the geoid. The geoid is usually described by the separation between itself and a mathematical surface. This separation is called geoidal height or geoidal undulation or Geoid separation (Aleem et al., 2013). The mathematical surface is a biaxial ellipsoid with the dimension chosen so that it describes the Earth's shape as closely as possible and it is the same ellipsoid that is used as a reference surface for measuring geodetic latitude and geodetic longitude.

Geoid is been described as equipotential surface which coincides on the average with the mean sea level has significant relevance in geodesy, surveying and other earth related disciplines. It finds applications in geographic information systems (GIS), engineering, the transformation of ellipsoidal heights of points to the orthometric heights, etc. This paper focuses on the geoid modelling technique based on geometrical interpolation approach by fitting a surface that depends on the reference points that are chosen in the critical and characteristic locations of the field to represent the trend of the geoid surface. Using the orthometric heights and the ellipsoidal heights, empirical geoidal undulations for all the points were computed. A multiple regression model was formulated as the required geometrical model to further adjust the derived geoid undulations from observation. Using a surface interpolation (Kriging) approach, the coordinate and the computed geoidal heights of some well selected points were utilized in Surfer 8 for gridding. This was used as a model for generating the geoidal heights of any other arbitrary points whose coordinates are known. From the analysis, it was observed that the use of the lower order polynomial (regression) to further model the geoid surface gave the mean square errors of 0.36cm and 5.58cm in self and cross validations respectively, with a smoothed geoid terrain, while the fundamental equation that relates the trio (orthometric, ellipsoidal and geoidal heights) gave the mean square errors of 7.35cm and 187.831cm in self and cross validations respectively. The modelled surface was generated at contour intervals of 0.5m for the two equations, while the digital terrain models were also generated in both cases (Nwilo, 2008),

Differential Global Positioning System (DGPS) is one of the most frequently used positioning methods in geodesy. The end products of surveying with Global Navigation Satellite Systems (GNSS) are geodetic latitude (ϕ), geodetic longitude (λ) and ellipsoidal height (h) which are obtained with reference to the ellipsoid. Recent developments in GNSS technology make us to obtain the ellipsoidal height with high accuracy. In engineering practice, orthometric heights (height above sea level) are always used. The orthometric heights are determined by spirit or geodetic levelling. In transforming the GNSS-derived ellipsoidal heights to orthometric heights, it is important to know the separation between the ellipsoidal and the geoid surface. This work investigates the use of ellipsoidal heights in place of orthometric heights for engineering surveys. DGPS observations were carried out to obtain the ellipsoidal heights for a number of points in the study area in Port Harcourt, Nigeria. Orthometric heights for the same set of points were determined using geodetic levelling. The results satisfied third order levelling which is good enough for engineering surveys (Badejo, 2016).

STATEMENT OF PROBLEM

In engineering works, engineers and surveyors are usually faced with the problem of determination of height differences between points. Some of the challenges are surveying of levelling networks, vertical applications, maintenance and control measurements of big

structures like bridges, dams, very tall buildings and towers, determination of crustal movements of the earth and motorways, railways, sewers and pipelines measurement. Instruments used in surveying and measurement methods are determined in relation to topography of land, target precision, and the aim. In this study, accuracies of heights determination techniques were based on the instrument and the measurement methods (Raaed, 2014). Digital technology for the production of geoidal map, surface modelling and terrain analysis is lacking in the field of Geomatics engineering. The major concern is lack of an existing unified geoidal map for surface modelling in geodetic, surveying applications, exploration and exploitation (Badejo, 2016). It is in light of the above that this research was carried out to produce a geoidal map through orthometric and ellipsoidal surface modeling of the study area. If this terrain analysis are well produced and presented, it would help in scientific investigations of mineral resource, in the determination of water flood volumes estimation, in soil erosion volumes estimation and engineering purpose.

AIM AND OBJECTIVES OF THE STUDY

The aim of this project is to analyses triple surface using dual frequency GPS and precise level. In order to achieve this aim, the following objectives were followed:

- i. To determine orthometric and ellipsoidal of the study area
- ii. To compute geoidal heights of the study area
- iii. To produce geoidal map, aspect map and slope map of the study area
- iv. To analyses the geoidal map, contour map, aspect map and slope map produce from the triple surfaces.

JUSTIFICATION OF THE STUDY

Considering the numerous problems of flooding and the need of contour map, geoidal map use for civil engineering work, environmental monitoring and control, with a comprehensive contour map, geoidal map, aspect map and slope map, the environmental monitory agency can easily identify the problematic areas most especially in the raining season in order to provide preventive measures for such occurrences. The products of this research will further be used for developing the area and assist in controlling future developmental plans in the state and the nation in general.

The fuel driving the engine of growth and sustainable development of any nation is the nation's access to reliable and sufficient geospatial information. In most developed countries, over 80 percent of rational and prospective allocation and environmental management decision is based on quality of accurate information (Ahmed, 2009). On the other hand, the roots of under development of third world countries, such as Nigeria emanated from a number of factors which include poor quality of data collection, organization and management practices; and, lack of adequate knowledge to develop the area and manage the environment in a sustainable manner. The consequences of all these are obvious from air and water pollution, environmental degradation, diseases and death

e.tc. These are the challenges of surveyors, environmental managers and any other specializations that deal with the management of environment in Bauchi metropolis and Nigeria as a whole. The geoid that was determined will be useful in geophysical exploration and for geodetic application. In general, the information provided will be used in all aspects of physical developments that concern land in the study area. For instance, for planning purposes, construction of civil engineering work, building engineering, design and construction of works, both on the surface and underground. The research can be use in water/flood volumes estimation, soil erosion volumes estimation, for geophysical exploration of natural resources, geodetic application and scientific investigations.

STUDY AREA

Bauchi has a total of 49, 359,001 square kilometres of land mass, which represents about 5.3 percent of Nigeria's total landmass. The state is one of the few States in Northern Nigeria that has two distinctive ecological zones; namely, the Sudan savannah and the Sahel savannah. The Sudan savannah covers the Southern part of the State, whereas the Sahel also known as the semi-desert vegetation covers the western and northern parts of the State, which is characterized by isolated strands of horny shrubs and sandy soils. On the other hand, the south-western part of the State is mountainous as a result of continuation of the Jos-Plateau while the Northern part is generally sandy as a result of the influence of the desert. Bauchi state is located between latitudes $09^{\circ} 30' N$ and $09^{\circ} 50' N$, north of the equator and longitudes $09^{\circ} 50' E$ and $10^{\circ} 20' E$, east of the Greenwich meridian.

MATERIAL AND METHODS

The methodology adopted for acquiring complete datasets for precise representation of complex surfaces is divided into two stages namely: field work and data processing. The former deals with the equipment setup and data collection, while the latter focused on data manipulation and processing. The basic data include Ellipsoidal heights and orthometric heights which were acquired using Dual Frequency Global Positioning System (DGPS) and precise level respectively. Geoidal heights were derived from the observed ellipsoidal heights and orthometric heights. Geoidal heights (Geoid undulation) were computed from the separation of the reference ellipsoid with the geoid surface measured along the ellipsoidal normal. The three dimensional coordinates from Dual frequency (spatial data sources) were interpolated by gridding using spline method of interpolation in ArcGIS 10.2.1 to produce the three dimensional surface model, Geoidal map and contour map of the study area. Geoidal, aspect and slope maps were therefore produced and the results were finally analysed and presented.

EQUIPMENT USED

The equipment's needed for the research work are as follows:

HARDWARE USED

- i. DGPS receiver and its accessories to acquire data for ellipsoidal height
- ii. Precise level and its accessories to acquire data for orthometric height
- iii. Computer and its accessories for computation, processing and analysis

SOFTWARE USED

- i. ArcGIS 10.2.1 software for interpolation
- ii. Microsoft Office excel 2007
- iii. Microsoft Office word 2007
- iv. Spectrum survey office (SSO)

METHOD

The factors considered in the reconnaissance include the design of the network and techniques adopted for effective execution of the work. In this research study, office and field reconnaissance were carried out. These are explained as follows;

Field reconnaissance was carried out to locate suitable positions for control establishment. In preliminary survey the existing control points adopted for connection were determined and in-situ was ascertained. The control points/benchmarks observed were established in the study area (good choice of stations marks). Site inspection played a vital role in facilitating the work and ascertaining the methods applied. The problems of intervisibility between the stations were completely avoided in case of precise levelling. The recce diagram of the study area was produced and control in-situ check was adopted on the existing control points used for connection of the DGPS observation to ascertain the quality of the original control used. Both the DGPS receiver and the precise level were tested on a control points and reference datum respectively, in order to ascertain their accuracy. The coordinates of the control points used for connection, the orthometric heights of the benchmarks and Bauchi State street guide map was collected from Bauchi State Ministry of lands and Housing. Booking sheet was designed and adopted for all field observations and measurements.

DATA ACQUISITION

The field operations were carried out for the purposes of acquiring the ellipsoidal heights and orthometric heights for a number of well distributed points in the research area. Precise levelling and DGPS field exercises were conducted in this work. The data captured includes latitude (ϕ), longitude (λ) and ellipsoidal heights (h) of all points of interest, using DGPS Receivers and Precise level for orthometric heights (H). The geoidal heights (N) was computed by subtracting orthometric heights from ellipsoidal heights. These sets of data were obtained from the site by means of direct field observation. The sample of the observed data is presented in Table 1.

Table 1: observed data

Longitude	Latitude	h	H	N
1134068.88	583691.59	656.080	612.010	44.070
1134087.40	583674.85	656.240	611.663	44.577
1134106.36	583658.41	656.464	611.979	44.485
1134124.89	583641.71	657.405	612.283	45.121

1134144.45	583626.07	657.514	613.254	44.260
1134163.61	583610.08	657.895	613.666	44.228
1134151.25	583614.02	658.322	614.191	44.131
1134135.00	583595.32	658.115	613.933	44.183
1134119.47	583575.83	658.194	613.966	44.228
1134102.40	583556.93	658.401	614.260	44.141
1134085.66	583538.75	657.906	613.791	44.115
1134073.77	583563.50	657.933	613.739	44.194
1134056.57	583579.67	657.688	613.604	44.084
1134033.90	583594.28	657.608	613.323	44.285
1134017.97	583613.19	656.910	612.840	44.070
1133994.89	583623.93	657.070	612.912	44.158
1133989.79	583633.71	656.305	612.144	44.160
1134011.04	583657.93	656.137	611.962	44.175
1134001.09	583671.27	656.383	611.965	44.418
1134031.39	583642.09	655.740	611.556	44.184
1134059.71	583616.90	656.358	612.088	44.270

MAP COMPILATION USING INTERPRETATION METHOD

The processed data (geoidal undulations) was used for the production of digital contour map of the study area. In this research work ArcGIS 10.2.1 was used for generating contour, geoidal map and surface analysis. The methods were grouped into smoothing and exact interpolators. Smoothing interpolators are: Inverse Distance to a Power, Kriging, Polynomial Regression, Radial Basis Function, Spline, Modified Shepard's Method, Local Polynomial, Moving Average; while the exact interpolators are: Inverse Distance to a Power, Kriging, Nearest Neighbor, Radial Basis Function, Modified Sheppard's Method, Triangulation with Linear Interpolation, and Natural Neighbor. But Spline methods of interpolation were used to approximate the geoid in the study area.

RESULT PRESENTATION

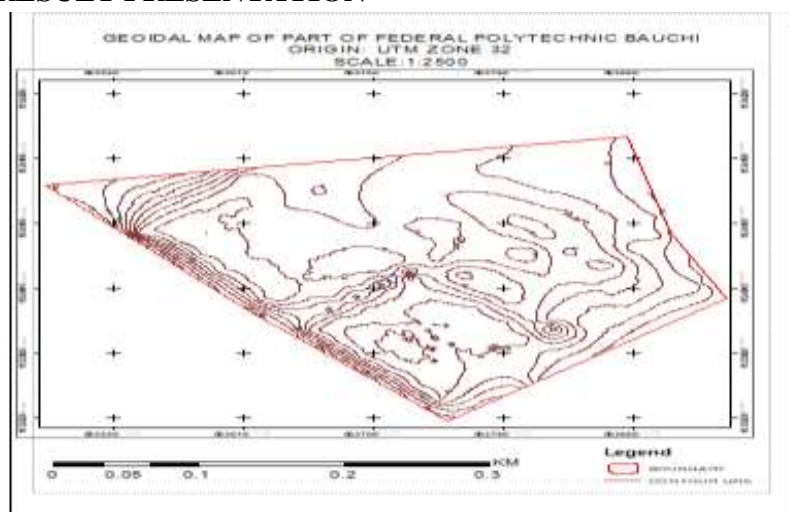


Figure 1: Geoidal map of the Study Area

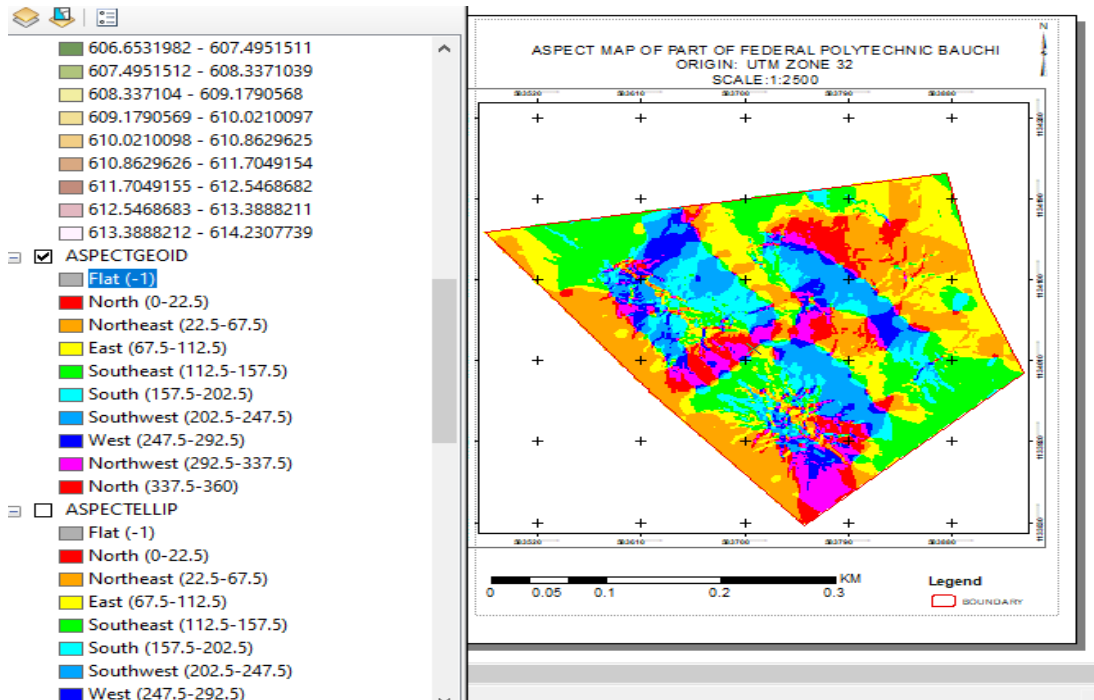


Figure 2: Aspect map of the Study Area using Geoidal heights

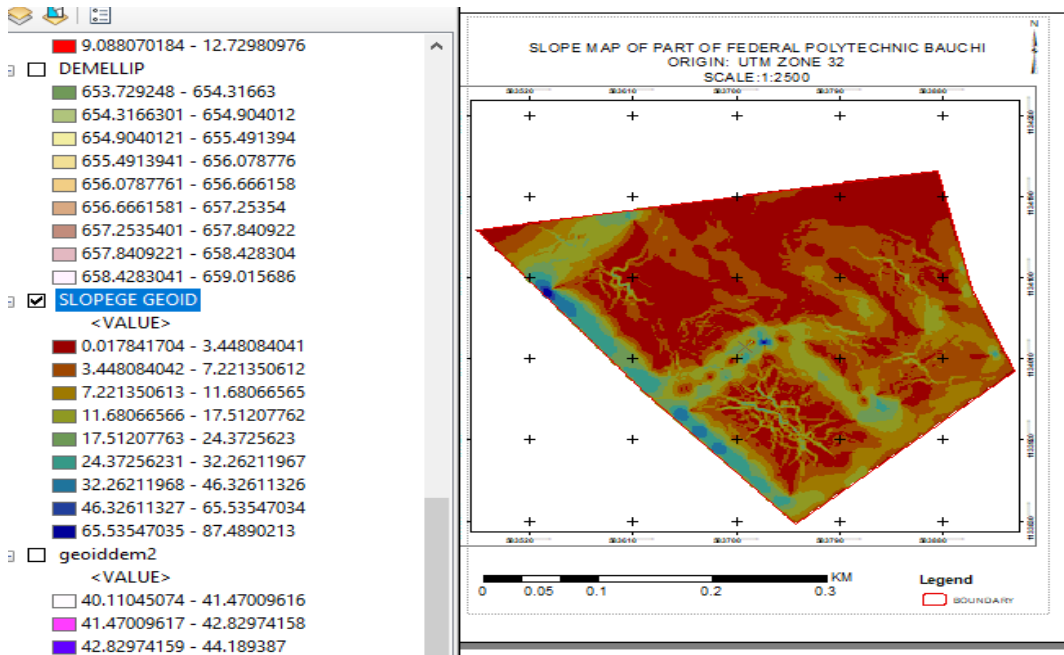


Figure 3: Slope map of the Study Area using Geoidal heights

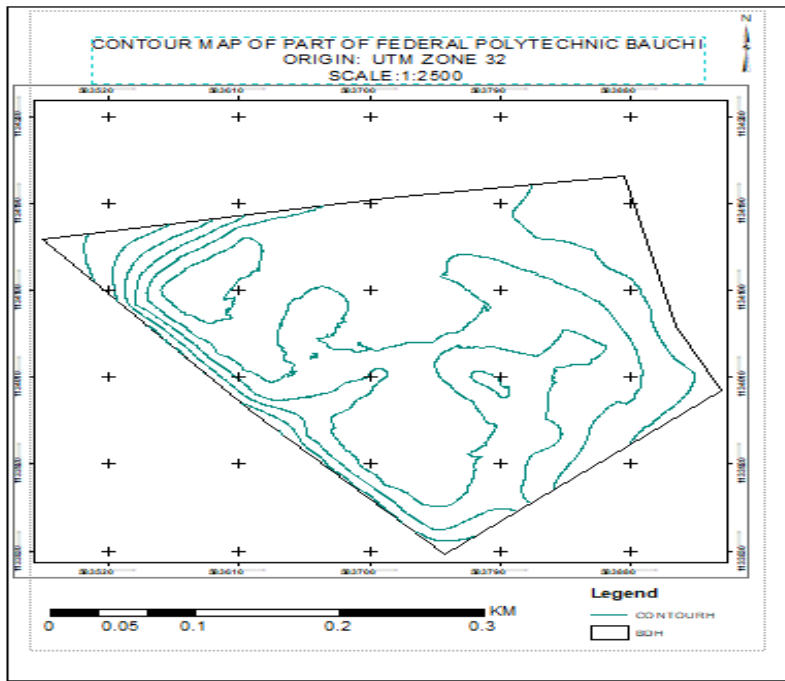


Figure 4: Contour map of the Study Area using orthometric heights

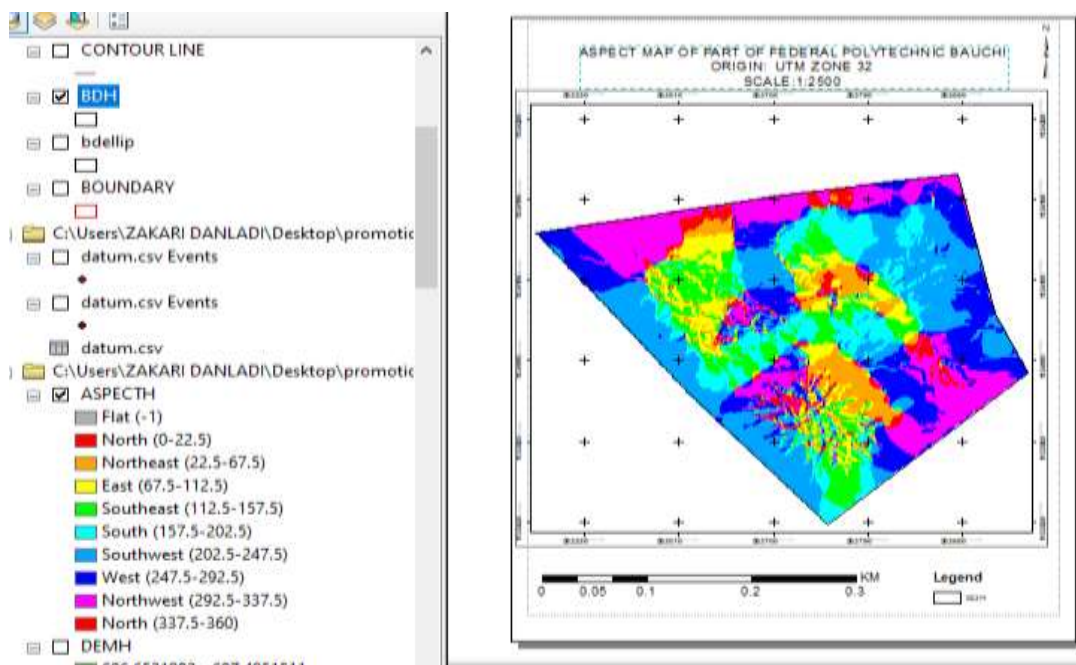


Figure 5: Aspect map of the Study Area using orthometric heights

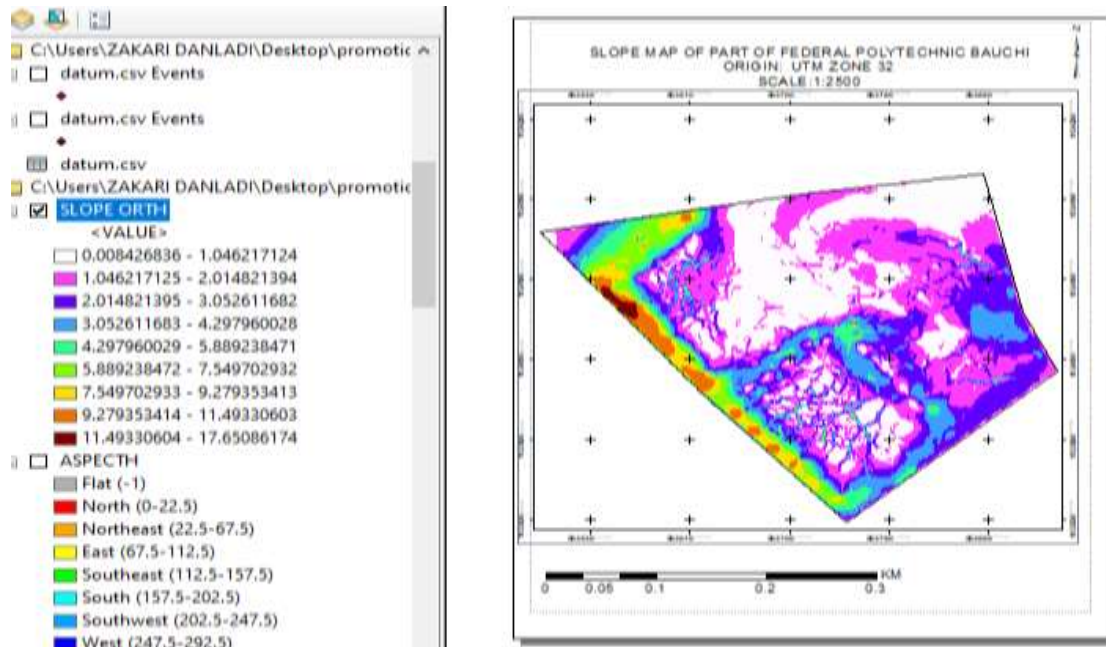


Figure 5: Slope map of the Study Area using orthometric heights

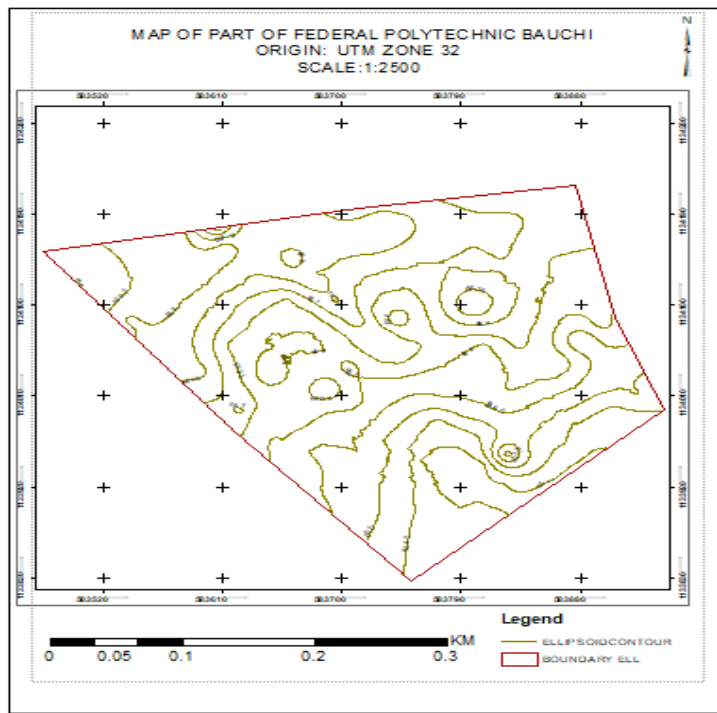


Figure 7: Contour map of the Study Area using Ellipsoidal heights

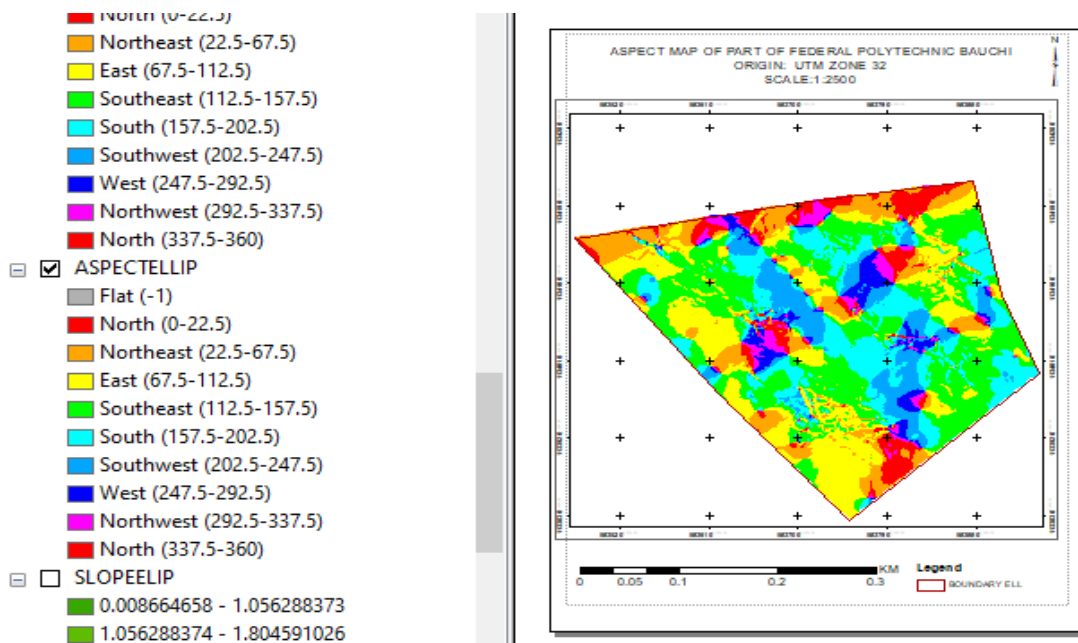


Figure 8: Aspect map of the Study Area using Ellipsoidal heights

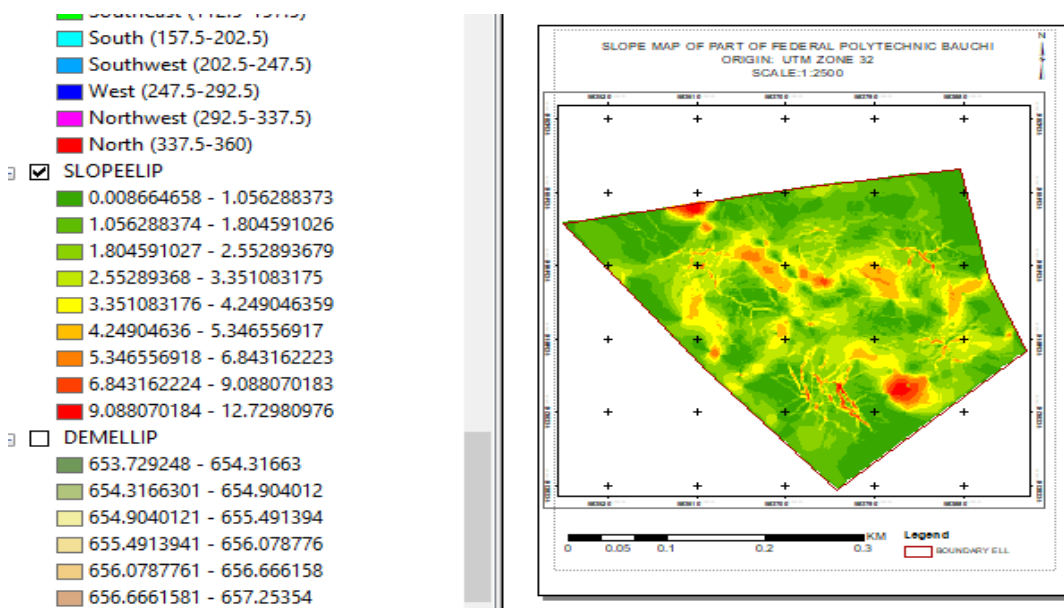


Figure 9: Slope map of the Study Area using Ellipsoidal heights

ANALYSIS OF RESULTS

Geoidal map was produced using the geoidal heights deduced from orthometric and ellipsoidal heights. The geoidal map portrayed the actual property on a geoid surface, which approximate mean sea level. The geoidal undulation at any point on the surface of the earth are points of different elevation. The geoid surface is not smooth but varies from

one point to another. This research indicated the the water level, that is the mean sea level within the study area is 44.560 meters below the physical surface which was as the most probable value of the observation. The triple map produced from geoidal heights are; geoidal map, aspect map and sope map as shown on Figure 1, Figure 2 and Figure 3 respectively.

The orthometric heights was used for the production of contour map, aspect map and slope map as presented on Figure 4, Figure 5 and Figure 6. The ellipsoidal heights was also used for the production of contour map, aspect map and slope map as presented on Figure 7, Figure 8 and Figure 9. The maps produced using both orthometric and ellipsoidal heights showed that there is not significance different on the two aspect map (Figure 5 and Figure 8). The triple slope map produced are invarriant to each other as indicated on Figure 3, Figure 6 and Figure 9. This is an indication that the physical surface, the geoid surface and the ellipsoidal surface are independent in their respective properties and they both portrayed difference in elevation between points. Also the the triple contour map generated behave differently, the contour map produced from geoid heights was completely different from the one produced from orthometric and ellipsoidal heights

SUMMARY

In the determination of Geoidal undulation for production of geoidal map, aspect map and slope map, the coordinates obtained from the field, ellipsoidal and orthometric heights were determined with the aid of Dual Frequency GPS receiver (DGPS) and precise Level Wild N3 instruments respectively. The heights determined with precise Levelling were reduced and orthometric heights were determined. The DGPS coordinates obtained and heights determined were post processed using the Spectrum survey offices software and the final adjusted coordinates and heights were determined. Geoidal heights of the study area were obtained from the differences between the orthometric and the ellipsoidal heights. The heights determined were exported from Microsoft office excel 2007 to ArcGIS 10.2.1 version. Shape files were created for each layer and were used in the production of the maps. The geoidal model was created using ArcGIS 10.2.1 version. The Geoidal heights determined were used in the compilation of the Geoidal map, aspect map and slope map of the study area. The orthometric and ellipsoidal heights were independently used in the production of the Contour map, aspect map and slope map of the study area.

CONCLUSION

In conclusion, levelled heights were established along with DGPS observations in the study area to unify the height system. The geoidal undulations were determined from the differences between orthometric and ellipsoidal heights. Geoidal heights were used for the production of Geoidal map of the study area. The three heights determined “geoidal, orthometric and ellipsoidal heights” were used in each case for the production of geoidal

map (Contour map), aspect map and slope map. The research showed that the triple surfaces (geoidal, orthometric and ellipsoidal surface) is independent of each other since each portrayed different elevation. On the three surfaces, the direction of flow of water is the same as indicated on the aspect map. There is a gentle slope as shown on the slope map generated from the geoidal map whereas on the physical surface, the slope is rapid as indicated on the slope map generated from orthometric and ellipsoidal heights.

RECOMMENDATIONS

In view of the foregoing results, it is therefore recommended that:

The management of the Federal Polytechnic Bauchi should use this research as a reference material for borehole construction and any other construction that has to do with the geoid or mean sea level.

Researcher, should use the geoidal heights determined to find out the local ground subsidence within the area, determine the ground stability in the case of structural movement or earthquake.

Further research work should use the geoidal heights determined to produce a unified mathematical model of the study area and the Nigeria as a whole.

REFERENCES

- Abd E. E. M. A. (2013): GPS ellipsoid height calibration to determine the approximate mean sea level (orthometric) height, international journal of advanced research in engineering and applied sciences, Vol. 2, No.8, pp 10-20
- Ahmed A. (2009): Determination of a gravimetric geoid model of Sudan using the KTH method, master's of science thesis in geodesy No. 3109 TRITA-GIT EX 09-01, Division of Geodesy Royal Institute of Technology, vol. 39, pp 21-27
- Ahmed A. S., Ali A. E., Abdalla A. S. and Ahmed M. Y. (2010): Obtaining Orthometric Heights With High Accuracy By GPS Observations Over Small Areas In Egypt, Surveying Dept. Shoubra Faculty of Engineering, Zagazig University Benha Branch, master of science thesis in Geodesy, No. 3109
- Aleem, K. F., Adesoye, A. A. and Bankole, A. L. (2016): Practical determination of geoidal undulation and geoidal map of part of Mubi, Adamawa state, Nigeria, International Journal of engineering research & technology (IJERT), ISSN: 2278-0181, Vol. 5, pp 740-747
- Anthony A. O., (2011): Determination of Nigerian geoid undulations from spherical harmonic analysis", Journal of applied physics Vol. 3, No. 1; Pp 74-77
- Amal M. A. (2016): Determination of the geoid height (geoid undulation) by using modern surveying technologies, applied research journal, Vol. 2, Pp.403-411
- Carlos A. (2009): Dissertation thesis in physical geodesy at institute of navigation and satellite geodesy graz university of technology, portugal, Departamento de Matemática da Faculdade de Ciências da Universidade de Lisboa Rua Ernesto Vasconcelos, Edifício C6, 1974-016 LISBOA. cmantunes@fc.ul.pt Vol. 2 | No. 8, pp 10-20
- Chen Y. And Yang Z. H. (2009): A Hybrid Method to determine the Hong Kong Geoid China, Department of Land Surveying and Geo-Informatics, The Hong Kong Polytechnic University, Hunghom, Portugal
- Dianah R. A., Roger H. and B. T. (2014): Evaluation of EGM2008 by means of GPS/Levelling Uganda, South African Journal of Geomatics, Vol. 3, No. 3 PP 272-284
- Edan J. D, Idowu, T. O, Abubakar T and Aliyu, M. R (2014) Determination of orthometric heights from GPS and levelling data, International Journal Of Electronics, Communication & Instrumentation Engineering Research and Development (IJECIERD) ISSN(P): 2249-684X; ISSN(E): 2249-7951, Vol. 4, Yola, Adamawa State, Nigeria.
- Erol, and Çelik R. N., (2004): Precise local geoid determination to make GPS technique more effective in practical applications of geodesy, FIG working week 2004, PP 22-27 May, Athens, Greece.
- Francis O. and Victor N. (2016), Determination of Best Fitting Geoid for Enugu State, Gravimetric Approach, Nigeria FIG Working Week 2016 Recovery from Disaster Christchurch, New Zealan

- Gomaa M. D. and Hoda F. M. (2009): Fitting Gravimetric Local and Global Quasi-Geoids, Survey Research Institute, National Water Research Center, Egypt, Vol. 20, pp: 47-59
- Hyo J. Y. (2014): Geoid Determination based on a Combination of Terrestrial and Airborne Gravity Data in South Korea, Dissertation Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio