



# NIGHTINGALE PUBLICATIONS AND RESEARCH INTERNATIONAL

## STABILITY ANALYSES OF TWO PARTIES IN DIRECT ELECTORAL COMPETITION

**ABDULKAREEM IBRAHIM AFOLABI,  
UMORU A, YAKUBU, NURUDEEN O. LASISI**  
*Federal Polytechnic Kaura-Namoda, Zamfara  
State, Nigeria.*

### Introduction

An ideal political democracy is the one that provides political opportunities for broad based electorates in deciding who acquire which political office based on popularity or proportionality ([Becker & Becker, 2013](#)). The central element in democratic theory is opportunity provided for competition between organized political parties through canvassing for electorates' votes employing different political tactics and strategies ([Ware, 1996](#)). The competitiveness of elections are determine by the obtainability of competitive parties, differences in their philosophical footings, and when the threshold of representation is lesser ([Lachat, 2011](#)).

One of the fundamental constitutive elements of a well-functioning democracy is the existence of meaningful competition among parties for elected positions. If strong opposition parties exist and maintain links with voters, they can keep the incumbent party in check by incentivizing them to perform better through the threat of electoral

### Abstract

*Electoral competitions are the instrument provided by democracy to ensure accountability and change of government. The continuous domination of one party in government overtakes might erode the significant of political democracy. In the light of this, we study the stability of two parties in direct electoral competition with aim of determine what determine the strength of these parties for effective electoral competition and as well their longtime co-existence. The dynamics of the model yields non-partisan and partisan equilibriums states. . The stability analysis of non-partisan yields an unstable scenario which might either leads to continuous nonpartisan electoral competition or gradual emergency of political awareness and subsequent*

*formation of political parties. Partisan equilibrium analysis suggested a complex spiral which always leads to oscillation in electoral success of the parties depending on their electoral reinforcement strategies. It is revealed from our findings that the parties will continue to be electorally relevant and co-exist as long as their electoral reinforcement strategies is effective to stimulate voters' interest*

**Keywords:** *Direct, Stability, Competition, Parent, Analyses*

---

defeat in subsequent elections. ([Liaqat et al., 2017](#)). Electoral competition between two strong political parties have ensure among others. It brings about societal representation in governance and in the legislative arena, and connect the government with civil society groups but they also stimulate other associational activities ([Basedau & Stroh, 2008](#); [Lipset, 2000](#); [Mair, 1997](#); [Pasquino, 2005](#)).

The political dynamics occasioned by electoral competition had received an erudite attention of mathematical researchers for decades. A kinetic model of bipartisan political system was proposed by Schiavo in ([Schiavo, 2006](#)). He assessed by simulation, the impact of social preference in the balloting pattern intern of party ideology or adherence. A unique mixed strategy Nash equilibrium in a one-dimensional Downsian model of two-candidate elections with a continuous policy space, where candidates are office motivated and one candidate enjoys a non-policy advantage over the other candidate was proposed by ([Aragonès & Xefteris, 2012](#)). They show that if voters' utility functions are concave and the median voter ideal point is drawn from a unimodal distribution, there is a mixed strategy Nash equilibrium where the advantaged candidate chooses the ideal point of the expected median voter with probability one and the disadvantaged candidate uses a mixed strategy that is symmetric around it.

A spatial voting model with proportional rule was proposed by ([De Sinopoli & Iannantuoni, 2007, 2008](#)) . They opened that if the outcome function is a linear combination of parties' positions, with coefficient equal to their shares of votes, essentially only a two-party equilibrium exist. A model of electoral competition where candidates selection policies and voting are determined by how much voters update their beliefs about their own private preferences after listening to arguments and then

vote in the election was proposed by (Hummel, 2012).. He showed that candidates adopt more divergent policies when voters are exposed to more arguments before the election.

Political parties unanimity Nash equilibrium occurs when the parties are strong enough not only to co-exist but also strong enough to contend with each other based on their popularity.(Roemer & Roemer, 2009). In the light of the need to have a credible democracy where incumbency accountability and popularity can be tested, we proposed in this work, a stability analysis of two parties in direct electoral competition with aim of strengthening good governance. In section 2, of this work we proposed a model of two parties in direct electoral competitions and perform stability analysis for emerging equilibrium states. In section 3, the discussion of the results with their political implications were ventilated.

### Model and Stability Analysis

#### The Model

Political literatures revealed that election success is based on how much a political party strives to canvass support in terms of recruitment of members and the consequential impact of her opponent(s) on their popularity before the electorate. In the light of this we proposed a competitive model of two parties as illustrated in equation (1).

$$\begin{aligned}\frac{dx}{dt} &= \omega_1 x(t) - \beta_1 x(t)y(t) \\ \frac{dy}{dt} &= \omega_2 y(t) - \beta_2 x(t)y(t)\end{aligned}\quad (1)$$

The first equation in (1) is the rate of change in electoral popularity of party-x over time. The first term  $\omega_1 x(t)$  is the electoral reinforcement strategies of party-x and the second term  $\beta_1 x(t)y(t)$  symbolizes the rate at which party-y upset the popularity of party-x. Similarly, the second equation of (1) is the rate of change in electoral popularity of party-y over time. The first term  $\omega_1 y(t)$  is the electoral reinforcement strategies of party-y and the second term  $\beta_2 x(t)y(t)$  symbolizes the rate at which party-x upsets the popularity of party-y.

Good governance hinges on effective electoral competitions which according to Nash equilibrium occurs when the parties are strong enough not only to

co-exist but also strong enough to contend with each other based on their popularity. We intend to obtain this equilibrium and its stability.

## 2.2 Equilibrium Point and Stability Analysis.

Equilibrium point here is a state where the two parties are strong enough to co-exist and remain unfettered by the presence of other. This is occurred at a point where the changes in popularity of one do not overwhelm the competitive strength of the others. This is mathematically obtained by setting equations in (1) to be equal to zero and obtained corresponding values for  $x$  and  $y$ . Then, the equilibrium points are  $(x^*, y^*) = (0, 0)$  a non-partisan equilibrium which occur when electoral competition is not party based and  $(x^*, y^*) = \left(\frac{\omega_2}{\beta_2}, \frac{\omega_1}{\beta_1}\right)$  a partisan equilibrium which occur when electoral competition is party based.

Linearizing equation (1), The Jacobin matrix takes the form:

$$J = \begin{pmatrix} \omega_1 - \beta_1 y^* & -\beta_1 x^* \\ \beta_2 y^* & \omega_2 - \beta_2 x^* \end{pmatrix} \quad (3)$$

The characteristic equation becomes.

$$\lambda^2 + (\beta_1 y^* + \beta_2 x^* - \omega_1 - \omega_2)\lambda + \beta_1 \beta_2 x^* y^* = 0 \quad (4)$$

### Stability Analysis of Non-Partisan Equilibrium

**Lemma 1:** The non-partisan equilibrium is always unstable.

**Proof:** Substituting  $(x^*, y^*) = (0, 0)$  into (4), we have.

$$\begin{aligned} \lambda^2 - (\omega_1 + \omega_2)\lambda &= 0 \\ \lambda_1 &= 0, \quad \lambda_2 = \omega_1 + \omega_2 \end{aligned} \quad (5)$$

The Eigen values associated with non-partisan equilibrium suggested an always unstable situation, which might swings between no party competition and emergences of leading contending political parties.

**Lemma 2:** The partisan Equilibrium is a complex spiral.

**Proof:** substituting  $(x^*, y^*) = \left(\frac{\omega_2}{\beta_2}, \frac{\omega_1}{\beta_1}\right)$  into (4), we have.

$$\begin{aligned} \lambda^2 + \omega_1 \omega_2 &= 0 \\ \lambda_{1,2} &= \pm i \sqrt{\omega_1 \omega_2} \end{aligned} \quad (6)$$

Thus, the partisan equilibrium point is a complex spiral which may leads to either growth or decay in popularity of the two parties depending on their reinforcement strategies.

## Conclusion

In this work, we proposed a model for two political parties in direct electoral competition. We studied the dynamics of the competition to arrival at two equilibriums state namely non-partisan and partisan equilibriums. The stability analysis of non-partisan yields an unstable scenario which might either leads to continuous nonpartisan electoral competition or gradual emergency of political awareness and subsequent formation of political parties. Stability analysis of partisan equilibrium is a complex spiral which always leads to oscillatory triumph of one of the parties depending on their electoral reinforcement strategies. It is revealed from our findings that electoral reinforcement strategies shape both the political alliance that leads to party formation in a non-partisan electoral competition, and the electoral success of the parties. That the parties will continue to be electorally relevant and co-exist as long as their electoral reinforcement strategies is effective to stimulate voters' interest.

## REFERENCES

- Aragonès, E., & Xefteris, D. (2012). Candidate quality in a Downsian model with a continuous policy space. *Games and Economic Behavior*, 75(2), 464-480.
- Basedau, M., & Stroh, A. (2008). Measuring party institutionalization in developing countries: A new research instrument applied to 28 African political parties.
- Becker, L. C., & Becker, C. B. (2013). *Encyclopedia of ethics*: Routledge.
- De Sinopoli, F., & Iannantuoni, G. (2007). A spatial voting model where proportional rule leads to two-party equilibria. *International Journal of Game Theory*, 35(2), 267-286.
- De Sinopoli, F., & Iannantuoni, G. (2008). Extreme voting under proportional representation: the multidimensional case. *Social Choice and Welfare*, 30(3), 401.
- Hummel, P. (2012). Deliberative democracy and electoral competition. *Games and Economic Behavior*, 75(2), 646-667.
- Lachat, R. (2011). Electoral competitiveness and issue voting. *Political Behavior*, 33(4), 645-663.

- Liaqat, A., Callen, M., Cheema, A., Khan, A. Q., Naseer, M. F., & Shapiro, J. N. (2017). The role of election competition in strengthening Pakistan's fledgling local democracy. *International Growth Centre Blog*.
- Lipset, S. M. (2000). The indispensability of political parties. *Journal of democracy*, 11(1), 48-55.
- Mair, P. (1997). EE schattschneider's the semisovereign people. *Political Studies*, 45(5), 947-954.
- Pasquino, G. (2005). The political science of Giovanni Sartori. *European Political Science*, 4(1), 33-41.
- Roemer, J. E., & Roemer, J. E. (2009). *Political competition: Theory and applications*: Harvard University Press.
- Schiavo, M. L. (2006). A dynamical model of electoral competition. *Mathematical and computer modelling*, 43(11-12), 1288-1309.
- Ware, A. (1996). *Political parties and party systems* (Vol. 9): Oxford University Press Oxford.