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MODELLING THE DIFFERENTIAL EFFECTS OF THE INITIAL CONDITION ON THE TYPE OF STABILIZATION

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Abstract

The deterministic stabilization of a dynamical system using the procedure of a simulation modelling is a challenging mathematical problem especially in the context of considering the differential effects of the initial condition on the qualitative characterization of a dynamical system

Keywords:
Condition, Initial, Stabilization, Modelling, Differrential.

INTRODUCTION

In the population modelling, it is a common practice to study the type of stabilization for a single specified initial condition depending on the context of the problem that is being investigated. We also know that initial condition values are only approximately estimated, therefore, it is important to evaluate how a deterministic stabilization of a dynamical system will respond to the differential effects of the initial condition. This idea is yet to be fully considered in the work of Yan and Ekaka-a (2011).

otherwise called stabilization. We have dominantly observed that the differential effects of changing the initial conditions have produced a robust similar type of stabilization in which each obtained steady state solution is uniformly stable. These novel results that we have obtained can be implemented to construct a feedback control to further stabilize this several stable steady state solution. The full results of this study are presented and discussed quantitatively.

Materials and Methods

The model parameter values that we have utilized to solve our proposed problem are provided by Pielou (1977) based on the pure competition mathematical description of two (2) yeast species with two (2) enhancing intrinsic growth rate parameters values of 0.1 and 0.08, followed by two inhibiting intra-competition coefficients of 0.0014 and 0.001 and two (2) inhibiting inter-competition coefficients of 0.012 and 0.0009 with the initial condition(4,10). In this study, we first found the median of this open interval to be seven (7). Then by fixing the first coordinate of this initial condition, and modifying the second coordinate, and similarly fixing the second coordinate and modifying the first coordinate, we have systematically studied the differential effects of the initial condition(4,10) variation on stabilization. Our core method of tackling this problem is called ordinary differential equation of order 45 (ODE 45) numerical scheme.

Results

The results that we have obtained in this study are as displayed in Table 1.1, Table 1.2, Table 2.1 and Table 2.2

Table 1.1: Evaluating the effect of changing the second coordinate of the initial condition (4, 10) on stability: Scenario One

EXAMPLE	IC*	x_e	y_e	λ_1	λ_2	TOS**
1	(4, 10)	12.499212271818017	68.746191385261781	-0.003310213400620	-0.082924684437944	Stable
2	(4, 7.0)	12.498998229665732	68.745188704999990	-0.003308710035310	-0.082922187270453	Stable
3	(4, 7.2)	2.495068407170804	68.727863702409252	-0.003282379851067	-0.082878537103175	Stable

4	(4, 7.4)	2.496022853960158	68.732033738257556	-0.003288730493898	-0.082889062028179	Stable
5	(4, 7.6)	2.496771293955041	68.735306557544135	-0.003293713742196	-0.082897321029579	Stable
6	(4, 7.8)	2.497360104363663	68.737883525165429	-0.003297636705566	-0.082903822961109	Stable
7	(4, 8.0)	2.497823638373131	68.739913971543942	-0.003300727071894	-0.082908945099027	Stable
8	(4, 8.2)	2.499752461938280	68.748503183504837	-0.003313750972515	-0.082930543323872	Stable
9	(4, 8.4)	2.499793148213740	68.748685592805813	-0.003314027138549	-0.082931001406821	Stable
10	(4, 8.6)	2.499821512447285	68.748814125896573	-0.003314221270164	-0.082931323528760	Stable
11	(4, 8.8)	2.499840116266739	68.748899968743416	-0.003314350404661	-0.082931537925505	Stable
12	(4, 9.0)	2.499850924130772	68.748951701408657	-0.003314427609670	-0.082931666254121	Stable
13	(4, 9.2)	2.498918507490767	68.744700680481785	-0.003308015710080	-0.082921024945177	Stable

***IC** represents the initial condition ****TOS** represents the type of stability

Table 1.2: Evaluating the effect of changing the second coordinate of the initial condition (4, 10) on stability: Scenario Two

EXAMPLE	**IC***	x_e	y_e	λ_1	λ_2	**TOS****
1	(4, 10)	12.499212271818017	68.746191385261781	-0.003310213400620	-0.082924684437944	Stable
2	(4, 9.4)	12.499001929273005	68.745070596095616	-0.003308577154844	-0.082921955890972	Stable
3	(4, 9.6)	12.499051702977729	68.745293313248041	-0.003308914495099	-0.082922515408313	Stable
4	(4, 9.8)	12.499073752091743	68.745394745799089	-0.003309067183600	-0.082922768885697	Stable
5	(4, 10.2)	12.499090137906254	68.745660890976410	-0.003309404414499	-0.082923343946879	Stable
6	(4, 10.4)	12.498952876232018	68.745064343465287	- 0.003308494820623	-0.082921836720524	Stable
7	(4, 10.6)	12.498800000553985	68.744399632919823	-0.003307481400739	-0.082920157426654	Stable
8	(4, 10.8)	12.498631002512409	68.743664551977105	- 0.003306360788329	-0.082918300487293	Stable
9	(4, 11.0)	12.498445460310633	68.742857270412316	-0.003305130191583	-0.082916261276886	Stable
10	(4, 11.2)	12.486596539517926	68.691136184471318	-0.003226345799322	-0.082785697187203	Stable
11	(4, 11.4)	12.488380743278219	68.698908185436252	-0.003238190317393	-0.082805324626132	Stable
12	(4, 11.6)	12.489882233526334	68.705455799705533	-0.003248166338557	-0.082821856484548	Stable
13	(4, 11.8)	12.491141903488291	68.710954381815242	-0.003256542093918	-0.082835736970797	Stable

***IC** represents the initial condition ****TOS** represents the type of stability

Table 2.1: Evaluating the effect of changing the first coordinate of the initial condition (4, 10) on stability: Scenario One

EXAMPLE	IC*	x_e	y_e	λ_1	λ_2	TOS**
1	(4, 10)	12.499212271818017	68.746191385261781	-0.003310213400620	-0.082924684437944	Stable
2	(7, 10)	12.496922911973531	68.736121214269801	-0.003294901123799	-0.082899301536166	Stable
3	(6.8, 10)	12.498700005344910	68.743874975119809	-0.003306713109182	-0.082918876830977	Stable
4	(6.6, 10)	12.499058757659474	68.745420130782477	-0.003309074033315	-0.082922787788529	Stable
5	(6.4, 10)	12.499668237311774	68.748107791538729	-0.003313158418341	-0.082929558992637	Stable
6	(6.2, 10)	12.499380870759582	68.746855630873597	-0.003311250314343	-0.082926396926262	Stable
7	(6.0, 10)	12.499329549379427	68.746636327355034	-0.003310914612046	-0.082925840968194	Stable
8	(5.8, 10)	12.499292655371015	68.746480111111296	-0.003310674967367	-0.082925444213062	Stable
9	(5.6, 10)	12.499266612687300	68.746371415658473	-0.003310507653849	-0.082925167343201	Stable
10	(5.4, 10)	12.499248599361071	68.746297948537844	-0.003310393938279	-0.082924979314678	Stable
11	(5.2, 10)	12.499236713608841	68.746251428851849	-0.003310321199759	-0.082924859212920	Stable
12	(5.0, 10)	12.499229763677819	68.746226673268225	-0.003310281536312	-0.082924793943754	Stable
13	(4.8, 10)	12.499226500741109	68.746218250671305	-0.003310266668058	-0.082924769786832	Stable

***IC** represents the initial condition ****TOS** represents the type of stability

Table 2.2: Evaluating the effect of changing the first coordinate of the initial condition (4, 10) on stability: scenario Two

EXAMPLE	IC*	x_e	y_e	λ_1	λ_2	TOS**
1	(4, 10)	12.499212271818017	68.746191385261781	-0.003310213400620	-0.082924684437944	Stable
2	(4.6, 10)	12.499235949418424	68.746266359233118	-0.003310337544094	-0.082924887818301	Stable
3	(4.4, 10)	12.499231624235842	68.746254198086845	-0.003310316666544	-0.082924853777007	Stable
4	(4.2, 10)	12.499229170464343	68.746250591671881	-0.003310308684658	-0.082924841139410	Stable
5	(3.8, 10)	12.499160699672402	68.745974105604873	-0.003309879688690	-0.082924132038033	Stable
6	(3.6, 10)	12.499103716522258	68.745733814615107	-0.003309510712149	-0.082923521245752	Stable
7	(3.4, 10)	12.499040913637431	68.745468832074252	-0.003309103875222	-0.082922847767874	Stable
8	(3.2, 10)	12.498972261593478	68.745179160904897	-0.003308659136332	-0.082922111546460	Stable
9	(3.0, 10)	12.498898428080846	68.744867883313447	-0.003308181131638	-0.082921320278864	Stable
10	(2.8, 10)	12.498821320811318	68.744543543442546	-0.003307682799881	-0.082920495426137	Stable
11	(2.6, 10)	12.498744998653239	68.744224143553296	-0.003307191462842	-0.082919682291546	Stable
12	(2.4, 10)	12.498677172115871	68.743943723001266	-0.003306758836419	-0.082918966614014	Stable
13	(2.2, 10)	12.498631604810619	68.743762888216182	-0.003306477053638	-0.082918501126453	Stable

***IC** represents the initial condition ****TOS** represents the type of stability

Discussion of Results

We hereby discuss the above results as follows:

From Table 1.1, when the initial condition is$(4,10)$, the optimal steady state solution is$(12.499212271818017, 68.746191385261781)$ which gives a value of approximately$(12.5, 68.7)$. As $t \to \infty$, we have observed that the deterministic steady state solution has two real negative eigenvalues, therefore, this optimal steady state solution is said to be stable, in which the two (2) negative eigenvalues contribute to the decaying behaviour of the solution trajectories. (This observation is consistent with related simulation analysis of stability in the works of Ekaka-a (2009) and Yan and Ekaka-a (2011). We have also observed from Table 1.1 to Table 1.2 that the obtained steady state solutions are dominantly stable when the first coordinate of the initial condition is changing while the second coordinate of the initial condition having the value of ten (10) is fixed. A similar observation can be seen when the first coordinate of the initial condition is changing while the second coordinate of the initial condition is fixed (Table 2.1 to Table 2.2).

Conclusion

The present study has made a significant contribution to knowledge in being able to identify the differential effects of the initial condition on stabilization, particularly for the competition interaction between two (2) yeast species. This novel contribution to knowledge is expected to provide a viable insight on the debate between biodiversity and ecosystem stability. The same numerical idea that we have utilized in this present paper can be extended to tackle a differential effect of the other model parameter values for a fixed initial condition value of (4,10) in or future investigation.

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