VOL. 17 NO. 8
ISSN: 2359-1991
MARCH, 2020

African Scholar Publications & Research International

African Scholars Journal of Contemporary Education Research (JCER-8)

Geometric Thought Processes as Correlates of Secondary School Students' Geometry Achievement in Taraba State

Musa Mohammed Balasa
Department of Mathematics, College of Education, P.M.B 1021 Zing, Taraba State.

Abstract
This investigation was carried out to determine relationship between students' thought processes (as measured by their thinking level scores) and their geometry achievement in Taraba State. A correlation design was used for the investigation because it was a study of relationship between variables. From a population of 14, 973 senior secondary two (SS2) students in the state of Taraba, a sample of 1, 170 was selected and used. Also, a sample of 12 out of 118 senior secondary schools was used. Cluster sampling method was used for both selections. Four research questions were asked to guide the conduct of the study and three null hypotheses were formulated and tested at the 0.05 significance level. Our instruments known as Concepts in Geometry Test (CGT), Geometry Items Sorting Test (GIST), Geometry Proof Test (GPT) and Van Hiele Geometry Test (VHGT) were used to collect data. Correlations were done between VHGT scores and CGT scores; VHGT scores and GIST scores; and VHGT scores and GPT scores. The t-test statistic (for significance of r) was used to determine the significance of r at the 0.05 level of significance. The results indicated the existence of strong, positive relationships between students' levels of geometric thinking and their achievement in geometry.

Keywords: *Geometry; thought process; achievement; senior secondary 2; thinking; thinking levels.*

Introduction

Any nation striving to develop economically must make corresponding advances in science and technology. Nigeria through its current vision plan called Nigeria vision 20-20, desires to become one of the twenty most developed economies of the world by the year 2020. A section of the vision document reads, "By 2020, Nigeria will be one of the 20 largest economies in the world, able to consolidate its leadership role in Africa and establish itself as a significant player in the global economic and political arena" (Federal Ministry of Education, 2005). To be among the world's 20 top-most economies, some of the conditions Nigeria needs to have on the ground, according to Ayagi (2008), include, among other things:

- having a reliable and a steady source of power generation, transmission and distribution;
- having a strong industrial base; and
- having improvements in agriculture, health, education, mining and other vital sectors of the national economy.

Clearly, all these conditions are hardly met without the continuous application of science and technology. Developments in science and technology are facilitated by having a mathematically literate citizenry. Wilson (2005) observed that the subject mathematics does not only aid the intellectual development of an individual, it is also the foundation upon which the much needed scientific and technological development of the individual's country stands. Mathematics is an effective tool for developing the capacities of individuals for clear logical thinking with a view to finding scientific solutions to problems. A branch of mathematics involving lots of critical and logical thinking is geometry. Hence the focus of this study on geometric thinking levels (thought processes) of students and their achievement in geometry.

The frequency with which the West African Examinations Council (WAEC) sets questions on topics in geometry is high, thus attracting investigations into the subject area of geometry. For instance, 35%, 38%, 40%, 42% and 37% of the multiple choice items prepared by WAEC for 2012, 2013, 2014, 2015 and 2016 respectively were all drawn from geometry. Further, WAEC chief examiner's report for the May/June 2015 West African School Certificate Examination included geometry among areas of weakness of candidates. WAEC'S act of setting questions frequently in geometry (WAEC, 2012, 2013, 2014, 2015, 2016) and its declaration of geometry as a difficult subject and one in which learners are generally weak (WAEC chief Examiner, 2015) are sources of inspiration for the researcher to focus on geometry.

To think means to use one's mind or to consider plans in one's mind (De Jager-Haum, 2000). Thinking therefore refers to using one's mind or considering plans in one's mind. Thinking is in levels according to Van Hiele (1986). Levels of thinking are the levels or stages at which thinking occurs in different sophistications. That is to say that thinking at level n is not as sophisticated as it is at level n+1. This brings us to hierarchy of thinking levels. Hierarchy of levels explains that levels of thinking are in a fixed sequence with some of the levels being lower or higher than others. For example, a child cannot operate at thinking level n+1 without first having gone through level n or, a child cannot be at thinking level n without first having been at level n-1, etc. Pierre Marie Van Hiele, a great Dutch Mathematics Educator, propounded and developed the geometric thinking level theory. He said, in their learning of school geometry, students' thought processes occur in stages which he called levels of thinking. He identified five of these levels but the current study concentrated on only four because even Van Hiele himself in his later writings, discarded the fifth level. The 4 levels are:

Level 1 (or recognition Level), 2. Level 2 (0r Analysis Level), 3. Level 3 (or order Level), and 4. Level 4 (or Formal Deduction Level). A prominent feature of these thinking levels is that a student operating at level n for instance may not learn the geometry that is associated with thinking level n+1. The present investigation explored the relationship between students' scores on these levels of thinking and their achievement in geometry.

The learning of school geometry in Nigeria has for a long time been based on the formal axiomatic geometry which Euclid created over 2000 years ago (Adele, cited in Atebe & Schafer, 2010). In his era, Euclid's logical construction of geometry with its axioms, postulates, definitions, theorems and proofs was, indeed, an admirable achievement (Van Hiele, 1999). However, Van Hiele (1999), expressed the view that school geometry presented and learnt in the traditional Euclidian fashion assumes, that school children also think at the formal deductive level (level 4). Van Hiele's argument here can be seen in the light of the following statement: For students to learn school geometry in the

Euclidean fashion, they should strive to not only understand but also translate axioms, postulates, definitions and theorems into constructible geometry.
But not all the students are at this formal deductive level. The thought processes injected into the learning of this kind of geometry may not be the same for all the students at the same time. Worried by students' weakness in their understanding of school geometry as reported by WAEC (2015) and by their poor performance in the subject, (WAEC, 2010, 2011), the current researcher embarked on the study of relationship between students' Van Hiele geometric thinking levels and their achievement in geometry.

Objectives of the Study

The purpose of this study was to determine the relationship between students' geometric thinking level scores (thought processes) and their achievement in geometry. Specifically, the study sought to determine the:

1. Van Hiele geometric thinking levels of senior secondary two (SS2) students in the state of Taraba;
2. Relationship existing between students' Van Hiele geometric thinking level scores and their achievement in Concepts in Geometry Test CGT);
3. Relationship existing between students' Van Hiele geometric thinking level scores and their achievement in Geometry Items Sorting Test (GIST); and
4. Relationship existing between students' Van Hiele geometric thinking level scores and their achievement in Geometry Proof Test (GPT).

Research Questions

The following research questions guided the conduct of the investigation:

1. What are the Van Hiele geometric thinking levels of senior secondary two (SS2) students in the state of Taraba?
2. What relationship exists between students' Van Hiele geometric thinking level scores and their achievement in Concepts in Geometry Test (CGT)?
3. What relationship exists between students' Van Hiele geometric thinking level scores and their achievement in Geometry Items Sorting Test (GIST)?
4. What relationship exists between students' Van Hiele geometric thinking level scores and their achievement in Geometry Proof Test (GPT)?

Research Hypotheses

The following hypotheses were formulated and tested at the 0.05 level of significance

H_{o1}. No significant relationship exists between students' Van Hiele geometric thinking level scores and their achievement in Concepts in Geometry Test (CGT).

H_{o2}. Students' Van Hiele geometric thinking level scores do not relate significantly with their achievement in Geometry Items Sorting Test (GIST).

H_{o3}. There is no significant relationship between students' Van Hiele Geometric thinking level scores and their achievement in Geometry Proof Test (GPT).

Method

A correlation design was used to investigate the relationship between students' geometric thinking levels and their achievement in three geometry tests Viz, Concepts in Geometry Test (CGT), Geometry Items Sorting Test (GIST), and Geometry Proof Test (GPT). The choice of the correlational design was informed by the idea that the study sought to determine the relationship between two variables: Geometric thinking levels of students on one hand and their achievement in geometry on the other. A sample of 1170 SS2 students, statistically determined by using Yaro Yamane's statistical formula for sample size taking, was used. Also, a sample of 12 out of 118 senior secondary schools was used. Cluster sampling technique was used for taking the samples. Four tests-Van Hiele Geometry Test (VHGT), Concepts in Geometry Test (CGT), Geometry Items Sorting Test (GIST) and Geometry Proof Test (GPT) formed instruments for data collection. The CGT was instrument which basically explored students' understanding of concepts in geometry; The Geometry Items Sorting Test (GIST), a hands-on activity test, created to the researcher a chance to observe and assess learners' understanding of and thinking about geometric shapes and their properties; The GPT enabled the researcher get a measure of research participants' ability to write a proof test; and The VHGT was an instrument which helped to determine the geometric thinking levels of participants. The four instruments were validated by seasoned science education teachers, one each in Modibbo Adama University (MAU), Yola; Taraba State

University (TASU), Jalingo, Federal University of Agriculture (FUA), Makurdi and Nnamdi Azikiwe University (NAU), Awka. Test retest reliability approach was used to estimate the reliability of the tests. Pearson's Products Moment Correlation Coefficient Method (PPMC) was used to get reliability coefficients of 0.88, 0.81, 0.78 and 0.85 for CGT, GIST, GPT and VHGT respectively.

Research Procedure

To obtain information on which investigational results and conclusions were based, participating students were given instructions in which the Van Hiele phase descriptors were used. Teachers in classrooms in each participating school were groomed and advised to use the checklist of Van Hiele phase descriptors in their teaching. For the sake of emphasis, the checklist is presented here:

i) Teacher introduces the topic by recognizing and building on learners' prior knowledge
ii) Teacher delays introduction of formal vocabulary and condones learners' use of common informal terms in the ensuring discussion
iii) Teacher asks questions that seek to clarify students' imprecise terminology and gradually introduces formal mathematical language
iv) Teacher creates an interactive learning environment and encourages learners to challenge, contest and negotiate meanings and solutions to mathematical problems
v) Teacher asks questions that steer students' thought toward the central idea being developed
vi) Teacher uses open-ended questions and encourages learners to seek their own solution strategies.
vii) Teacher encourages learners to elaborate on their responses and
viii) Teacher uses questions that encourage learners to reflect on, refine and summarize their ideas about the concepts learned. To properly handle the research, the researcher organized a two-week training exercise for participating teachers. The research conditions were properly explained. The frame work for thinking about the tasks (the Van Hiele phase descriptors) were exhaustively explained to the teachers.

At the end of four weeks of instructions with participating SS2 students, the tests were administered. Students' scripts were marked and scored using the marking guide. Data collected were analyzed using the mean and standard deviation for answering the research questions, product moment correlation coefficient method to determine relationship between the variables and the t-test statistic (for significance of r) to test the significance of the hypotheses.

Result

This section is concerned with presentation and statistical analysis of results. The results are presented in tables. A research question precedes each table and the content of the table answers the research question. A hypothesis follows the research question for testing at the 5% level of significance. The Pearson's r and the t-test statistic (for significance of r) were used for the analyses. A statement follows the summary of each result, rejecting or accepting the stated hypothesis

Research Question 1:

What are the Van Hiele geometric thinking levels of senior secondary two (SS2) students in the state of Taraba?

Table 1 below indicates the Van Hiele levels of geometric thinking of participating senior secondary 2 students;

Table 1: Van Hiele Levels of Geometric Thinking of Participating SS2 students:

Levels Attained					
Schools	*1*	*2*	*3*	*4*	*Total*
1	*28*	*42*	*20*	*15*	*105*
2	*39*	*32*	*18*	*13*	*102*
3	*29*	*40*	*18*	*8*	*95*
4	*32*	*38*	*21*	*9*	*100*
5	*36*	*34*	*18*	*12*	*100*
6	*40*	*38*	*16*	*7*	*101*
7	*29*	*32*	*25*	*6*	*92*
8	*23*	*39*	*25*	*10*	*97*
9	*40*	*36*	*26*	*13*	*115*
10	*26*	*20*	*24*	*10*	*80*

11	*30*	*29*	*26*	*12*	*97*
12	*25*	*39*	*13*	*9*	*86*
Total	*377*	*419*	*250*	*124*	*1170*

For clearer interpretation, results in table 1 are summarized in table 2 below:

Table 2: Summary of SS2 students' Van Hiele Levels of Geometric Thinking

Levels Attained	*Number*	*Percentage*
1	*377*	*32.22*
2	*419*	*35.81*
3	*250*	*21.37*
4	*124*	*10.60*
Total	*1170*	*100%*

From table 2, it can be seen that the lowest Van Hiele level (level 1) was attained by 377 SS2 students (i.e. 32.22%) and the next lowest level (level 2) was attained by 419 SS2 students (i.e. 35.81%). A total of 796 SS2 students, representing 68.03% of the participating SS2 students, could only operate at the lower Van Hiele levels (Levels 1 and 2). The remaining 374 students, representing just 31.97% actually attained higher Van Hiele levels (3 and 4). Only 124 students (i.e. 10.6%) attained the highest Van Hiele level (i.e. level 4).

Students' Achievement in the VHGT according to Van Hiele Levels:

Presentation of students' achievement in Van Hiele Geometry Test (VHGT) is up next. The presentation is shown in table 3, indicating levels attained by these students and their achievement according to these levels.

Table 3: Students' Achievement in the VHGT According to Levels Attained:

Levels	*No*	*Total Scores*	*Mean Scores*	*Highest Scores*	*Lowest Scores*
1	*377*	*377*	*1.0*	*2*	*1*
2	*419*	*2367*	*5.65*	*4*	*2*
3	*250*	*1434*	*5.74*	*7*	*4*
4	*124*	*1217*	*9.81*	*13*	*8*
Total	*1170*				

From table 3, it is obviously seen that the lowest mean score of the participating students on the VHGT occurs at the lowest Van Hiele thinking level (level1), while the highest mean score on the same test occurred at the highest Van Hiele thinking level i.e. level 4. Further, the mean score of students who attained level 1, for instance, (1.0) is lower than that of those who attained level 2 (5.65). Again, the mean score of students on level 2 (5.65) is lower than that of those on level 3 (5.74) which is in turn lower than the mean score of students on level 4 (9.81). This means that the lower the level at which a student operates, the lower his/her mean score on a test. Similarly, the higher the student's level of operation, the higher his/her mean score on a test.

Table 4: Mean Ratings of the Van Hiele Geometric Thinking Levels of Senior Secondary Two (SS2) students in Taraba State.

S/N	*Test*	*N*	*Mean*	*Standard Deviation*
1	*VHGT*	*1170*	*4.07*	*3.66*
2	*CGT*	*1170*	*4.58*	*2.34*
3	*GIST*	*1170*	*4.45*	*2.36*
4	*GPT*	*1170*	*4.00*	*2.38*

VHGT = Van Hiele Geometry Test; CGT = Concepts in Geometry Test; GIST = Geometry Items Sorting Test; GPT = Geometry Proof Test.

Research Question 2:

What relationship exists between SS2 students' Van Hiele geometric thinking level scores and their achievement in Concepts in Geometry Test (CGT)?

The data for answering research question 2 are presented in table 5.

Table 5: Relationship between SS2 students' Van Hiele Geometric Thinking Level Scores and their Achievement in Concepts in Geometry Test (CGT):

Tests	*Mean*	*Standard Deviation*	*N*
VHGT	*4.0658*	*3.66426*	*1170*
CGT	*4.5829*	*2.33588*	*1170*

Correlation Matrix

		VHGT	CGT
VHGT	Pearson Correlation	1	.865*
	Sig. (2-tailed)		.000
	N	1170	1170
CGT	Pearson Correlation	.865*	1
	Sig. (2-tailed)	.000	
	N	1170	1170

**Correlation is significant at the 0.05 level (2-tailed)

Data in table 5 show that the calculated r (correlation coefficient) is 0. 87 which indicates that there is a high, positive relationship between senior secondary 2 students' Van Hiele Geometric Thinking Level Scores and their Achievement in Concepts in Geometry Test (CGT). The value of r also means that 75.69% of the variance observed on the students' achievement in the CGT was accounted for by their VHGT levels.

Hypothesis 1

No significant relationship exists between students' Van Hiele geometric thinking level scores and their achievement in Concepts in Geometry Test (CGT).

The data for testing hypothesis 1 are presented in table 6

Table 6: t-test statistic of the correlation between students' Van Hiele Geometric Thinking Level Scores and their Achievement in Concepts in Geometry Test (CGT)

Tests	N	Mean	Std. Dev	Std. Error Mean	t-cal	P-value	Decision
VHGT	1170	4.07	3.66	.14497	-2.264	.024	Significant
CGT	1170	4.58	2.34	.15838			

Data in table 6 indicate a P-value of 0.02 which is less than the alpha value of .05. This shows that the students' Van Hiele Geometric Thinking Level Scores statistically significantly relate with their achievement in Concepts in Geometry Test. Therefore, the hypothesis that no significant relationship exists between students' Van Hiele geometric thinking level scores and their achievement in Concepts in Geometry Test was rejected.

Research Question 3:

What relationship exists between students' Van Hiele geometric thinking level scores and their achievement in Geometry Items Sorting Test (GIST)?

The data for answering research question 3 are presented in table 7.

Table 7: Relationship between students' Van Hiele Geometric Thinking Level Scores and their Achievement in Geometry Items Sorting Test (GIST)

Tests	*Mean*	*Std. Deviation*	*N*
VHGT	*4.07*	*3.66*	*1170*
GIST	*4.45*	*2.36*	*1170*

Correlation Matrix

		VHGT	*GIST*
	Pearson Correlation	*1*	*.858**
VHGT	*Sig. (2-tailed)*		*.000*
	N	*1170*	*1170*
	Pearson Correlation	*.858**	*1*
	Sig. (2-tailed)	*.000*	
GIST	*N*	*1170*	*1170*

*Correlation is significant at the 0.05 level (2-tailed)

Data in table 7 reveal that the calculated r is .86 which indicates that there is a strong, positive relationship between students' Van Hiele geometric thinking level scores and their achievement in Geometry Items Sorting Test. The value of r also means that 73.96% of the variance observed on the students' achievement in the GIST was accounted for by their Van Hiele geometric thinking levels.

Hypothesis 2:

Students' Van Hiele Geometric Thinking Level Scores do not relate significantly with their achievement in Geometry Items Sorting Test (GIST)

The data for testing hypothesis 2 are presented in table 8.

Table 8: t-Test statistic of the correlation between students' Van Hiele Geometric Thinking Level Scores and their Achievement in Geometry Items Sorting Test (GIST)

Tests	N	Mean	Std. Dev	Std. Error Mean	t-cal	P-value	Decision
VHGT	1170	4.07	3.66	0.09212	-.536	.000	Significant
GIST	1170	4.45	2.36	0.09896			

Data in table 8 indicate a P-value of .00 which is lower than alpha value of .05. This means that the relationship between students' Van Hiele geometric thinking level scores and their achievement in Geometry Items Sorting Test was statistically significant. Therefore, the hypothesis of no significant relationship between students' Van Hiele geometric thinking level scores and their achievement in Geometry Items Sorting Test (GIST) was rejected.

Research Question 4:

What relationship exists between students' Van Hiele geometric thinking level scores and their achievement in Geometry Proof Test (GPT)?

The data for answering research question 4 are presented in table 9.

Table 9: Relationship between students' Van Hiele Geometric Thinking Level Scores and their Achievement in Geometry Proof Test (GPT)

Tests	*Mean*	*Std. Deviation*	*N*
VHGT	*4.07*	*3.66*	*1170*
GPT	*4.00*	*2.38*	*1170*

Correlation Matrix

		VHGT	*GPT*
VHGT	*Pearson Correlation*	*1*	*.861**
	Sig. (2-tailed)		*.000*
	N	*1170*	*1170*
	Pearson Correlation	*.858**	*1*
	Sig. (2-tailed)	*.000*	
GPT	*N*	*1170*	*1170*

Correlation Coefficient is significant at the 0.05 level (2-tailed)

Data in table 9 reveal that the correlation coefficient ® is .86 which indicates that students' Van Hiele Geometric Thinking Level Scores positively, strongly relate with their achievement in Geometry Proof Test (GPT). The value of r also means that 73.96% of the variance noticed on the students' achievement in Geometry Proof Test (GPT) was accounted for by their Van Hiele Geometry thinking levels.

Hypothesis 3

There is no significant relationship between students' Van Hiele Geometric thinking level scores and their achievement in Geometry Proof Test (GPT). The data for testing hypothesis 3 are presented in table 10.

Table 10: t-Test statistic of the correlation between students' Van Hiele Geometric Thinking Level Scores and their Achievement in Geometry Proof Test (GPT).

Tests	N	Mean	Std. Dev	Std. Error Mean	t-cal	P-value	Decision
VHGT	1170	4.07	3.66	0.09434	-4.463	.000	Significant
GPT	1170	4.00	2.38	0.09903			

Data in table 10 indicate a P-value of .00 which is less than the alpha value of .05. This implies that the relationship between students' Van Hiele Geometric thinking level scores and their achievement in Geometry Proof Test (GPT) was statistically significant. Hence the hypothesis of no significant relationship between students' Van Hiele geometric thinking level scores and their achievement in Geometry Proof Test (GPT) was rejected.

Summary of Findings

It was found from the study that:

1. A total of 796 participating SS2 students (representing 68.03%) were operating on the lower Van Hiele levels of geometric thinking (levels 1 and 2) while only 374 students (representing just 31.97%) operated on higher Van Hiele levels (levels 3 and 4), and only 124 students (i.e. 10.6%) attained the highest Van Hiele level (i.e. level 4).
2. There is a high, positive relationship between students' Van Hiele geometric thinking level scores and their achievement in Concepts in

Geometry Test, and students' Van Hiele geometric thinking level scores statistically significantly relate with their achievement in CGT

3. There is a strong, positive relationship between students' Van Hiele geometric thinking level scores and their achievement in GIST, and the relationship between students' VHGT level scores and their achievement in GIST was statistically significant
4. Students' VHGT scores positively, strongly relate with their achievement in GPT, and the relationship between students' VHGT level scores and their achievement in GPT was statistically significant.

Discussion

It was clear from the study that the geometric thinking levels were not only the determinants of content of geometry students could learn but also the level of achievement they could make in the investigational tests. For instance, it was discovered that majority of students in the study area (i.e. 796 students, representing 68.03%) operated on lower Van Hiele geometric thinking levels and could only learn content of geometry meant for those lower levels (1 and 2). This result is consistent with that of Usiskin cited in Atebe and Schafer (2010) which reported that, "An overall low percentage mean score of 35.68% attained by the learners was considered to be evidence that the majority of the learners in the study were at a low Van Hiele geometric thinking level, possibly "0 or 1". Unfortunately, classroom practices in the current study area are such that students are taught the same content as if they belong to the same thinking levels. This has far-reaching implications for the teacher.

A discovery was made from this study that the lowest score (1) on the VHGT occurred at the lowest VHGT level (level 1) while the highest score (13) occurred at the highest VHGT level (level 4). This means that students on lower VHGT level cannot achieve as much as students on higher VHGT level. Again, this has far-reaching implications for education. Atebe and Schafer (2010), had similar findings.

Another discovery made by the study was that of existence of statistically significant relationship between students' VHGT scores and their achievement in Concepts in Geometry Test (CGT). The finding here is again similar to that of Usiskin in Atebe and Schafer (2010), who used the CDASSG to measure students' attainment of the Van Hiele levels. However, Tieng and Kwan Eu

(2015), conducted a study on improving students' Van Hiele levels of geometric thinking using geometer sketchpad where they found no correlation between students' literacy in using Information and Communications Technology (ICT) and their Van Hiele levels of geometric thinking. Therefore, the discovery made by these researchers is not in harmony with that of current researcher.

Yet another discovery made by current study was that of existence of strong, positive relationship between students' Van Hiele thinking levels and their achievement in Geometry Proof Test, and that such relationship was statistically significant. This result confirms a previous finding by Senk (2009) that a positive correlation exists between Van Hiele levels and proof writing abilities of students. Also, the result is in line with Aydin and Halat (2009) in a study on the impacts of undergraduate mathematics courses on college students' geometric reasoning stages, which investigated possible effects of different college level mathematics courses on college students' Van Hiele levels of geometric understanding. The results showed that students taking logic/proof based courses attained higher reasoning stages than students taking other college level mathematics courses, such as calculus.

Educational Implications of the Findings

The findings of this research have far-reaching implications for education. They have implications for students, teachers, educators and curriculum planners. In the first place, it was found from the research that majority of sampled students (68.03%) merely acquired lower thinking levels (levels 1 and 2). Only 31.97% acquired higher thinking levels (3 and 4). It was equally noted from the literature that hierarchy of thinking levels is such that a student operating on level 1 for instance, may not learn the geometry content at level 2. Similarly, a student on level 2, may not learn the geometry content at level 3 etc. The implication of this for the classroom is that teaching and learning of school geometry should be level-specific. That is to say that those students on thinking level 1 should strictly learn content meant for this level. The same thing is expected for all other levels. Unfortunately, the practice in the study area is such that students are taught the same content of geometry in total disregard for their levels of thinking. In other words, students are made to learn the same geometric content as if they belong to the same geometric thinking levels. Learning problems can be expected from such a practice as some learners are made to learn content for which they are not ready.

The generally poor achievement of learners in Concepts in Geometry Test (CGT) was an indication of weakness in the knowledge of terminology in geometry. This holds an important implication for classroom teaching and learning. This is because, Van Hiele (1986) stresses the point that when a child has learned to recognize a figure by direct contact with it, he/she should develop the appropriate technical term or language with which to communicate ideas about the figure to others.

Quite a good number of students in this study had ability to recognize shapes only in some standard orientations. By implication, these learners did not understand that simple geometric shapes are defined by their properties and not by their orientations or positions in space. Teachers need to provide learners with activities for exploring the properties of geometric shapes in many and varied orientations.

Further, the findings said no to the general feeling among our population that our secondary school students can hardly solve difficult problems in geometry. Contrary to this feeling, many of the responses to such problems (attempted by students who luckily answered questions from level specific content), showed mature, systematic, step-by-step presentation and analytical reasoning. The meaning of this is that our students have the capability of solving difficult geometrical problems provided such problems are level-specific.

Conclusion

It was concluded from the research that:

1. Majority of SS2 students in the study area were operating on lower Van Hiele thinking levels (levels 1 and 2);
2. A strong, positive relationship exists between students' thinking levels and their achievement in Concepts in Geometry Test (CGT) and such relationship is statistically significant;
3. Students' geometric thinking levels are significantly related to their achievement in Geometry Items Sorting Test (GIST);
4. There is statistically significant relationship between students' thinking levels and their achievement in Geometry Proof Test (GPT).

Recommendations

Based on findings, the following recommendations were made:

1. Teacher should ensure that students learn content which matches their geometric thinking levels;
2. Teachers should determine their students' levels of geometric thinking before the commencement of academic programs so as to ensure that the students learn level specific content of geometry;
3. Curriculum planners should see to the possibility of re-designing the school geometry curriculum such that content will suit or match students' geometric thinking levels;
4. In designing and delivering instructions, teachers should not only teach the terminology associated with a given content area in geometry, they should also ensure that learners who have a correct verbal description of a geometric concept, also have a correct concept image associated with that concept and vice versa;
5. Teachers need to prepare and present a variety of activities that create numerous opportunities for learners to explore the properties of simple geometric shapes in many and varied orientations;
6. Since defining a geometric concept is only feasible if a learner knows to some extent the thing that is to be defined, (Van Hiele, 1986), teachers should note that learners need preliminary explorations of the properties of a shape before attempting to write a formal definition of it;
7. Students should use findings of this study to adjust their Van Hiele geometric thinking levels so as to enhance their learning and achievement in school geometry;
8. Geometry educators should use the findings of this study to properly select, organize and present materials to geometry teachers who in turn will be better informed as to how to select level-specific content and suitable instructional methods;
9. Curriculum planners should consider replanning the geometry curriculum such that content and sequencing fit the thinking levels of students;

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