NUMERICAL SIMULATIONS OF A DETERMINISTIC COMPARTMENTAL MODEL OF ZIKAVIRUS DISEASE DYNAMICS WITH CONTROLS.

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Abstract
In this study, a deterministic model of Zika virus disease dynamics with controls was presented. Parameters for treatment and use of insecticides were incorporated. The model equations were solved using Homotopy perturbation method.

Keywords: Zika virus, Control Parameters, Homotopy Perturbation, Insecticides and Treatment

Graphical profiles of each compartment were generated from the results using maple software. The result of the numerical

INTRODUCTION
Zika virus disease is a member of the family called Flaviviridae and the genus Flavivirus. (Musso et al2015). World health organization WHO has concluded that Zika virus infection during pregnancy is a cause of congenital brain abnormalities, including microcephaly; and that infection is a trigger of Guillain-Barre syndrome. Intense efforts are continuing to investigate the link between infectious virus and a range of neurological disorders, within a rigorous research framework (WHO, 2016). According to Kucharski et al, (2016) several lines of proof agree that the infection transmission is causing symptoms such as fever and rash. The infection has also been linked to increased incidence of neurological disorder, including Guillain-Barre Syndrome and microcephaly in infants.
simulation indicates that Strategy to control the treatment and the use of disease. insecticides is the best

born to mothers who were infected with infectious virus during pregnancy. About 80% of human infectivity develops mild disease consisting of mild fever, skin rashes, arthralgia, particularly swelling, lymphadenopathy, malaise or headache, conjunctivitis (Red eyes), muscle and joint pain, which last for 2-7 days normally, with no documented fatalities seen in a recent large outbreak. The microcephaly morbidity rate of the new baby is very high if the pregnancy was infected by the disease. And some evidence suggests that it causes Guillain-Barre syndrome (GBS) as well. According to the US Centre for Disease Control (CDC). Infectivity of the disease is through mosquito bites, it is transmitted to people primarily through the bite of an infected Aedes species mosquito. It can be transmitted from mother to child; a pregnant woman can pass the virus to her fetus during pregnancy (Centre for Disease Control, 2016). The infection is the cause of microcephaly and other severe brain defects. The infectious virus can remain in semen longer than in other body fluids, including vaginal fluids, urine, and blood. However, the transmission through blood transfusion, were reports of laboratory acquired Zika virus infections, although the route of transmission was not clearly established in all cases (Musso et al, 2015)

Infection with Zika virus may be suspected based on symptoms and recent history of travel. A diagnosis of the virus infection can only be confirmed through laboratory tests on blood or other body fluids, such as urine, saliva or semen (CDC, 2016).

For now, there is no vaccine for prevention of the disease. However, it can be prevented by avoiding mosquito bites through the use of bed nets and the use of insecticide spray and proper sanitation. The use of Condoms can reduce the chance of getting the virus from sex. In the case of blood transfusion, bloods should be screen before it is been transmitted to humans (CDC, 2016).

Unfortunately, no vaccine, specific treatment, or fast diagnostic test is available to treat, prevent, or diagnose ZIKV infection at this time. There is no specific Treatment for the disease yet but apparent symptoms are normally mild and can be treated with common pain and fever medicines, bed rest and taking plenty of water and healthy fluids (Khalid & Khan 2016)

LITERATURE REVIEW

Ding et al (2016) formulated a compartmental mathematical model according to the transmission rules in order to investigate the interior mechanism of ZIKA
virus transmission. Model results illustrate the use of basic reproduction number, the optimal control strategies are suggested to provide and control the disease. In their model, recruitment rate of mosquitoes, probability of both human and mosquitoes getting infected, natural death of both mosquitoes and human, death due to infection and the recovered class was not also considered. Bonyah and Okosun (2016) formulated a model of ZIKA, virus with its optimal control strategies. In their models, they proposed optimal control to examine the effect of mass treatment and insecticide. The purpose of this control is to minimize the number of infected host and vectors with optimal cost of mass treatment and insecticide. The global stability of disease-free equilibrium was found to be asymptotically stable. Optimal control strategies of prevention, treatment and insecticide are incorporated into the model. However, the model does not consider death due to the infection of ZIKA virus.

The model:

Colombia (Korzeniewski et al., 2016). Developed a mathematical model of transmission dynamics of Zika virus. The model examined 2013-14 outbreak of the virus infection in French Polynesia, the results suggested that the infection transmission in island population may follow similar pattern to dengue fever, generating large, sporadic outbreak with high degree of under-reporting. However, in their model recruitment rate of mosquitoes, natural death of human, natural death of mosquitoes and also death due to the infection of ZIKA virus infection was not considered.

Momoh and Fugenschuh (2017) developed an optimal control strategy and a cost effectiveness analysis for the Zika virus disease, they considered four preventive measures as control strategy: the use of treated bed nets, the use of condoms, a medical treatment of infected persons, and the use of indoor residual spray. They obtained the reproduction number for the model and carried out a stability analysis, they also examine the implementation of various combinations of the controls in order to determine the most cost-effective strategy among all the control strategies considered, based on the computational results obtained, they concluded that a strategy based on treated bed nets, treatment of infected individuals, and indoor residual spray is the most cost effective of all their examined strategies for a control of the Zika virus. However in their models death due to Zika virus infection was not considered.

Onuorah et al., (2016) developed a deterministic model describing the dynamics of ZIKA virus; a set of ordinary differential equation was developed. The basic reproductive number ($R_0$) of the model which is an important threshold for
disease control was obtained. Disease free equilibrium ($E_0$) which is stable whenever $R_0 < 1$, this means that one infected individual introduced in to an environment where everyone is susceptible is effecting less than one person, in which case, the virus will die out. Further analysis of the model shows that the endemic equilibrium exists and the conditions for its stability are given. However death due to Zika virus infection was not considered.

**MATERIALS AND METHODS**

We formulate a mathematical model of Zika virus infection. The model incorporated the use of control strategies (Treatment and insecticides) where Treatment is represented by $u_t$ and insecticides by $u_i$. The model contains two populations, which are human and Aedes mosquitoes with three and two compartments respectively. We examine the susceptible-infected-recovered model to explore the dynamics of transmission of Zika to human. The three compartments, namely: susceptible human ($S_H$), infected human ($I_H$) and the recovered human ($R_H$). The other two compartments for aedes mosquitoes are susceptible aedes mosquitoes ($S_A$) and infected aedes ($I_A$).

This leads to the following SIR-SI model.

**MODEL EQUATIONS**

\[
\frac{dS_H}{dt} = \Lambda_H - (\beta_H + \phi_H I_H + \mu_H)S_H
\]

(3.1)

\[
\frac{dI_H}{dt} = (\beta_H + \phi_H I_H)S_H - (\mu_H + \gamma_H + u_i)I_H
\]

(3.2)

\[
\frac{dR_H}{dt} = (\gamma_H + u_i)I_H - \mu_H R_H
\]

(3.3)

\[
\frac{dS_A}{dt} = \Lambda_A - (\beta_A + \phi_A I_H + \mu_A + u_i)S_A
\]

(3.4)

\[
\frac{dI_A}{dt} = \beta_A I_H S_A - (\mu_A + u_i)I_A
\]

(3.5)

With variables and parameters defined as follows

$\Lambda_H$  
Recruitment rate of human.

$\Lambda_A$  
Recruitment rate of Aedes mosquitoes.

$S_H$  
Susceptible individuals.
$I_H$ Infected individuals.
$R_H$ Recovered individual.
$S_A$ Susceptible Aedes mosquitoes.
$I_A$ Infected Aedes mosquitoes.
$\beta$ Transmission probability of human getting infected
$\beta_1$ Transmission probability of Aedes mosquitoes getting infected
$\varepsilon$ Per capital biting rate.
$\phi$ Contact rate of vector per human.
$\phi_1$ Transmission probability after sexual interaction between a susceptible human and infected human.
$\gamma_H$ Natural recovery.
$\mu_H$ Natural death rate of human
$\mu_A$ Natural death of Aedes mosquitoes
$\mu_0$ Death due to Zika virus infection

**ANALYTICAL SOLUTION OF THE MODEL USING HOMOTOPY PERTURBATION METHOD (HPM)**

The fundamental of Homotopy Perturbation Method (HPM) was first proposed by Ji-Haun (2000). The Homotopy Perturbation Method (HPM), which provides analytical approximate solution, is applied to various linear and non-linear equations. The homotopy perturbation method (HPM) is a series expansion method used in the solution of nonlinear partial differential equations Jiya (2010).

To show the simple concepts of this method, he considered the following non-linear differential equation:

$$A_3(U) - f(r) = 0, \quad r \in \Omega$$

(3.6)

Subject to the boundary condition

$$B_3\left(U, \frac{\partial U}{\partial n}\right) = 0, \quad r \in \Gamma$$

(3.7)

where $A_3$ is a general differential operator, $B_3$ a boundary operator, $f(r)$ is a known analytical function and $\Gamma$ is the boundary of the domain $\Omega$. The operator
A_3 can be divided into two parts L and N, where L is the linear part, and N is the nonlinear part. Equation (3.6) can be written as:

\[ L(U) + N(U) - f(r) = 0, \quad r \in \Omega \]  

(3.8)

The Homotopy Perturbation structure is shown as follows

\[ H(V, h) = (1 - h)[L(V) - L(U_0)] + h[A(V) - f(r)] = 0 \]  

(3.9)

where \( V(r, P); \Omega \in [0,1] \rightarrow R \)

In equation (3.8) \( P \in [0,1] \) is an embedding parameter and \( U_0 \) is the approximation that satisfies the boundary condition. It can be assumed that the solution of the equation (3.8) can be written as power series in \( h \) as follows:

\[ V = V_0 + hV_1 + h^2V_2 + \ldots \]  

(3.10)

And the best approximation for the solution is:

\[ U = \lim_{h \to 1} v_0 + hv_1 + h^2v_2 + \ldots \]  

(3.11)

The series (3.11) is convergent for most cases. However, the convergent rate depends on the nonlinear operator \( A(V) \)

### 3.3 SOLUTION OF THE MODEL EQUATIONS

\[
\frac{dS_H}{dt} + (\beta \mu I_A + \phi I_H + \mu_H)S_H - \Lambda_H = 0
\]  

(3.12)

\[
\frac{dI_H}{dt} + (\mu_H + \mu_0 + \gamma_H + u_1)I_H - (\beta \mu I_A + \phi I_H)S_H = 0
\]  

(3.13)

\[
\frac{dR_H}{dt} + \mu_H + R_H - (\gamma_H + u_1)I_H = 0
\]  

(3.14)

\[
\frac{dS_A}{dt} + (\beta I_H + \mu_A + u_2)S_A - \Lambda_A = 0
\]  

(3.15)

\[
\frac{dI_A}{dt} + (\mu_A + u_2)I_A - \beta I_H S_A = 0
\]  

(3.16)

With the following initial conditions

\[ S_H(0) = S_{H(0)}, \quad I_H(0) = I_{H(0)}, \quad R_H(0) = R_{H(0)}, \quad S_A(0) = S_{A(0)}, \quad I_A(0) = I_{A(0)} \]
Let
\[ S_H = a_o + h a_1 + h^2 a_2 + ..... \]  
(3.17)
\[ I_H = b_o + h b_1 + h^2 b_2 + ..... \]  
(3.18)
\[ R_H = c_o + h c_1 + h^2 c_2 + ..... \]  
(3.19)
\[ S_A = d_o + h d_1 + h^2 d_2 + ..... \]  
(3.20)
\[ I_A = e_o + h e_1 + h^2 e_2 + ..... \]  
(3.21)

Applying HPM to (3.12)
\[
(1-h) \frac{dS_H}{dt} + h \left[ \frac{dS_H}{dt} + (\beta e \phi I_A + \phi I_H + \mu_H)S_H - \Lambda_H \right] = 0
\]  
(3.22)

Substituting equation (3.17),(3.18) and (3.21) in to equation (3.12)
\[
(1-h)(a_o + h a_1 + h^2 a_2 + .....) + h \left[ (\beta e \phi (b_o + h b_1 + h^2 b_2 + .....)) + \phi (b_o + h b_1 + h^2 b_2 + .....)) + \right. \left. (\beta e \phi (e_o + h e_1 + h^2 e_2) \right) \right) + \mu_H (a_o + h a_1 + h^2 a_2 + ...) - \Lambda_H = 0
\]  
(3.23)

Expanding and collecting the coefficient of the powers of \( h \), we have
\[ h^0 : a_o' = 0 \]  
(3.24)
\[ h^1 : a_1' + \beta e \phi b_o + \phi I_H + \mu_H + a_o - \Lambda_H = 0 \]  
(3.25)
\[ h^2 : a_2 + b_1 \beta e \phi + \phi b_o + a_1 = 0 \]  
(3.26)

Applying (HPM) to (3.13)
\[
(1-h) \frac{dI_H}{dt} + h \left[ \frac{dI_H}{dt} + (\mu_H + \mu_o + \gamma_H + u_i)I_H - (\beta e \phi I_A + \phi I_H) \right] S_H = 0
\]  
(3.27)

Substituting equation (3.18),(3.21) and (3.17) in to equation (3.27)
\[
(1-h)(b_o + h b_1 + h^2 b_2 + ...) + h \left[ (h b_o + h^2 b_2 + ...) \right) - \left( \beta e \phi (e_o + h e_1 + h^2 e_2) \right) + \phi (b_o + h b_1 + h^2 b_2 + (...) \left( a_o + h a_i + h^2 a_2 + ... \right) \right) = 0
\]  
(3.28)

Expanding and collecting the coefficients of the powers of \( h \), we have
\[ h^0 : b_o' = 0 \]  
(3.29)
\[ h^1 : b_1' + (\mu_H + \mu_o + \gamma_H + u_i) b_o - \beta e \phi e_0 + \phi b_o + a_o = 0 \]  
(3.30)
\[ h^2 : b_2 + (b_1 (\mu_H + \mu_o + \gamma_H + u_i) b_o) = 0 \]  
(3.31)

Applying HPM to (3.14)
(1 – h) \frac{dR_H}{dt} + h \left[ \frac{dR_H}{dt} + \mu_H + R_H - (\gamma_H + u_1)I_H \right] = 0 \\
(3.32)

Substituting equation (3.19) and (3.18) in to equation (3.32)

(1 – h) \left[ \left( c_o' + h c_i + h^2 c_{i2} + .... \right) + \left( c_o' + h c_i + h^2 c_{i2} + .... \right) \right] = 0 \\
(3.33) 

Expanding and collecting the coefficient of the powers of h, we have

\begin{align*}
    h^0 : c_o' &= 0 \\
    h^1 : c_i + \mu_H c_o - (\gamma_H + u_1) b_o &= 0 \\
    h^2 : c_{i2} + \mu_H c_i + (\gamma_H + u_1) b_i &= 0 \\
\end{align*}

(3.34), (3.35), (3.36)

Applying HMP to (3.15)

(1 – h) \frac{dS_A}{dt} + h \left[ \frac{dS_A}{dt} + (\beta_i \varepsilon \phi I_A + (\mu_A + u_2)) S_A - \Lambda_A \right] = 0 \\
(3.37)

Substituting equation (3.20) and (3.21) in to equation (3.37)

Expanding and collecting of powers of h, we have

\begin{align*}
    h^0 : d_o' &= 0 \\
    h^1 : d_i + \beta_1 \varepsilon \phi e_o + \mu_A + u_2 + d_o &= 0 \\
    h^2 : d_{i2} + e_i \beta_1 \phi + \mu_A + u_2 + d_i &= 0 \\
\end{align*}

(3.38), (3.39), (3.40)

Applying HMP to (3.16)

(1 – h) \frac{dI_A}{dt} + h \left[ \frac{dI_A}{dt} + (\mu_A + u_2) I_A - \beta_1 \varepsilon \phi I_H S_A \right] = 0 \\
(3.41)

Substituting equation (3.21), (3.18) and (3.20) in to equation (3.41)

\begin{align*}
    (1 – h) \left( e_o' + h e_i + h^2 e_{i2} + .... \right) + h \left( e_o' + h e_i + h^2 a_{i2} + .... \right) \\
    \left( \mu_A + u_2 \left( e_o' + h e_i + h^2 e_{i2} + .... \right) \right) \left( \beta_1 \varepsilon \phi \left( b_o + h b_i + h^2 b_{i2} + .... \right) \right) \\
    \left( a_o + h a_i + h^2 a_{i2} + .... \right) &= 0 \\
\end{align*}

(3.42)

Expanding and collecting the coefficient of the powers of h, we have

\begin{align*}
    h^0 : e_o' &= 0 \\
    h^1 : e_i' + \left( \mu_A + u_2 \right) e_o - \beta_1 \varepsilon \phi b_o + a_o &= 0 \\
    h^2 : e_{i2} + e_i \left( \mu_A + u_2 \right) - \beta_1 \varepsilon \phi b_i + a_i &= 0 \\
\end{align*}

(3.43), (3.44), (3.45)

From equation (3.24)
\[ a_0' = 0 \]  \hspace{1cm} (3.46)

Integrating both sides and then applying the initial condition
\[ a_0(0) = S_{H(0)} = a_0 \] \hspace{1cm} (3.47)

we have
\[ a_0 = S_{H(0)} \] \hspace{1cm} (3.48)

From (3.29)
\[ b_0' = 0 \] \hspace{1cm} (3.49)

Integrating and applying the initial condition
\[ b_0(0) = I_{H(0)} = b_0 \] \hspace{1cm} (3.50)

We have
\[ b_0 = I_{H(0)} \] \hspace{1cm} (3.51)

From (3.34)
\[ h^o : c_0 = 0 \] \hspace{1cm} (3.52)

Integrating (3.126) and applying the initial condition
\[ c_0(0) = R_{H(0)} = c_0 \] \hspace{1cm} (3.53)

We have
\[ c_0 = R_{H(0)} \] \hspace{1cm} (3.54)

From equation (3.38)
\[ h^o : d^1_o = 0 \] \hspace{1cm} (3.55)

Integrating equation (3.129) and applying the initial condition
\[ d_0(0) = S_{A(0)} = d_0 \] \hspace{1cm} (3.56)

i.e
\[ d_0 = S_{A(0)} \] \hspace{1cm} (3.57)

From (3.43)
\[ h^o : e_0 = 0 \] \hspace{1cm} (3.58)

Integrating equation and applying the initial condition
\[ e_0(0) = I_{A(0)} = e_0 \] \hspace{1cm} (3.59)

i.e
\[ e_0 = I_{A(0)} \] \hspace{1cm} (3.60)

From equation (3.25)
\[ h^1: a^\prime_i + \beta \varepsilon \phi b_o + \phi_1 b_o + \mu_H + a_o - \Lambda_H = 0 \]  
\[ a^\prime_i = \Lambda_H - \beta \varepsilon \phi b_o - \phi_1 b_o - \mu_H \]  

Integrating (3.62) and applying the initial condition \( a_i(0) = 0 \)
\[ a_i = (\Lambda_H - \beta \varepsilon \phi b_o - \phi_1 b_o - \mu_H) t \]  

Put equation (3.51) in to equation (3.63)
\[ a_i = \left( \Lambda_H - \beta \varepsilon \phi I_H(0) - \phi I_H(0) - \mu_H \right) t \]  

From equation (3.30)
\[ h^1: b^\prime_i + (\mu_H + \mu_o + \gamma_H + u_i) b_o - \beta \varepsilon \phi e_o + \phi b_o + a_o = 0 \]  
\[ b^\prime_i = \beta \varepsilon \phi e_o - a_o - \phi_1 b_o - (\mu_H + \mu_o + \gamma_H + u_i) b_o \]  

Integrating equation (3.66) and applying the initial condition \( b_i(0) = 0 \)
\[ b_i = \left( \beta \varepsilon \phi e_o - a_o - \phi b_o - (\mu_H + \mu_o + \gamma_H + u_i) b_o \right) t \]  

Substituting equation (3.60), (3.48) and (3.52) in to (3.67)
\[ b_i = \left( \beta \varepsilon \phi R_H(0) - S_H(0) - \phi I_H(0) - (\mu_H + \mu_o + \gamma_H + u_i) \right) I_H(0) t \]  

From equation (3.35)
\[ h^1: c^\prime_i + \mu_H c_o - (\gamma_H + u_i) b_o = 0 \]  
\[ c^\prime_i = (\gamma_H + u_i) b_o - \mu_H c_0 = 0 \]  

Integrating equation (3.70) and applying the initial condition \( c_i(0) = 0 \)
\[ c_i = ((\gamma_H + u_i) b_o - \mu_H c_o) t \]  

Substituting equation (3.51) and (3.54) in to equation (3.71)
\[ c_i = \left( (\gamma_H + u_i) I_H(0) - \mu_H R_H(0) \right) t \]  

From equation (3.39)
\[ h^1: d^\prime_i + \beta \varepsilon \phi e_o + (\mu_A + u_2) + d_o = 0 \]  
\[ d^\prime_i = (\mu_A + u_2) - d_o - \beta \varepsilon \phi e_o = 0 \]  

Integrating equation (3.74) and applying the initial condition \( d_i(0) = 0 \)
\[ d_i = \left( (\mu_A + u_2) - d_o - \beta \varepsilon \phi e_o \right) t \]  

Substituting equation (3.57) and (3.60) in to (3.75)
\[ d_i = \left( (\mu_A + u_2) - S_A(0) - \beta \varepsilon \phi I_A(0) \right) t \]  

From equation (3.44)
\[ h^1: e^\prime_i + (\mu_A + u_2) e_o - \beta \varepsilon \phi b_o + a_o = 0 \]  

(3.61)  
(3.62)  
(3.63)  
(3.64)  
(3.65)  
(3.66)  
(3.67)  
(3.68)  
(3.69)  
(3.70)  
(3.71)  
(3.72)  
(3.73)  
(3.74)  
(3.75)  
(3.76)  
(3.77)
\[ e'_1 = (a_0 - \mu_A + u_2)e_0 + \beta_1 \epsilon \phi b_0 = 0 \]  
(3.78)

Integrating equation (3.78) and applying the initial condition \( e_1(0) = 0 \)
\[ e_1 = (a_0 - \mu_A + u_2)e_0 + \beta_1 \epsilon \phi b_0 \]
(3.79)

Substituting equation (3.48),(3.60) and (3.51) in to equation (3.79)
\[ e_1 = \left( S_{H(0)} - \mu_A + u_2 I_{A(0)} + \beta_1 \epsilon \phi I_{H(0)} \right) t \]
(3.80)

From equation (3.26)
\[ h^2 : a_2 + b_1 \beta \epsilon \phi + \phi b_1 + a_1 = 0 \]
(3.81)
\[ a_2 = -b_1 \beta \epsilon \phi - \phi b_1 - a_1 \]
(3.82)

Put equation (3.68) and (3.63) in to equation
\[ a_2 = \left[ -(\beta \epsilon \phi + \phi_1) \left( \epsilon \phi R_{H(0)} - S_{H(0)} - \phi I_{H(0)} - \mu_H + \mu_0 + \gamma_H + u_i \right) \right] t \]
(3.83)

Integrating (3.83) and applying the initial condition \( a_2(0) = 0 \), we have
\[ a_2 = \left[ \left( \beta \epsilon \phi R_{H(0)} - S_{H(0)} - \phi I_{H(0)} - \mu_H + \mu_0 + \gamma_H + u_i \right) \right] t^2/2 \]
(3.84)

Substitute equation (3.48), (3.64) and (3.84) in to equation (3.17) and then take these limit as \( h \rightarrow 1 \)
\[ S_H = S_{H(0)} + \left( \Lambda_H - \beta \epsilon \phi I_{H(0)} - \phi I_{H(0)} - \mu_H \right) t + \left( \beta \epsilon \phi + \phi_1 \right) \]
\[ \left( \beta \epsilon \phi R_{H(0)} - S_{H(0)} - \phi I_{H(0)} - \mu_H + \mu_0 + \gamma_H + u_i \right) \]
\[ \left( \left( \Lambda_H - \beta \epsilon \phi I_{H(0)} - \phi I_{H(0)} - \mu_H \right) \right] t^2 \]
(3.85)

From equation (3.31),
\[ h^2 : b_2 + \left( b_1 (\mu_H + \mu_0 + \gamma_H + u_i) \right) b_1 - \left( e_i \beta \epsilon \phi + \phi b_1 + a_1 \right) = 0 \]
(3.86)
\[ b_2 = \left( e_i \beta \epsilon \phi + \phi b_1 + a_1 \right) + \left( b_1 (\mu_H + \mu_0 + \gamma_H + u_i) \right) = 0 \]
(3.87)

\[ b_2 = \left[ \left( S_{H(0)} - \mu_A + u_2 \right) I_{A(0)} + \beta \epsilon \phi I_{H(0)} \right] \beta \epsilon \phi + \]
\[ \left( \beta \epsilon \phi R_{H(0)} - S_{H(0)} - \phi I_{H(0)} - \mu_H + \mu_0 + \gamma_H + u_i \right) \]
\[ \left( \Lambda_H - \beta \epsilon \phi I_{H(0)} - \phi I_{H(0)} - \mu_H \right) \]
\[ \left( \beta \epsilon \phi R_{H(0)} - S_{H(0)} - \phi I_{H(0)} - \mu_H + \mu_0 + \gamma_H + u_i \right) \]
(3.88)
Integrating equation (3.88) and applying the initial condition \( b_2(0) = 0 \), we have

\[
\begin{align*}
&\left[ (S_{H(0)} - (\mu_A + u_2) I_{A(0)} + \beta_e \varphi I_{H(0)}) + \beta_e \varphi \left( \beta_e \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} \right) + \left( \Lambda_H - \beta_e \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H - \left( \mu_H + \mu_0 + \gamma_H + u_1 \right) \right) \right]
\end{align*}
\]

\[
= \frac{t^2}{2}
\]

(3.89)

Substitute equation (3.51), (3.68) and (3.89) into (3.18) and then take the limit as \( h \to 1 \)

\[
I_H = I_{H(0)} + \left( \beta_e \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - \left( \mu_H + \mu_0 + \gamma_H + u_1 \right) \right) t + \left( \Lambda_H - \beta_e \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H - \left( \mu_H + \mu_0 + \gamma_H + u_1 \right) \right)
\]

\[
= \frac{t^2}{2}
\]

(3.90)

From equation (3.36),

\[
h^2 : c_1' + (\mu_H + c_1 + (\gamma_H + u_1))b_1 = 0
\]

(3.91)

\[
c_1' = (\gamma_H + u_1) b_1 - \mu_H c_1
\]

(3.92)

Put equation (3.68) and (3.72) into equation (3.92)

\[
c_2 = \left[ \gamma_H + u_1 \left( \left( \beta_e \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - \left( \mu_H + \mu_0 + \gamma_H + u_1 \right) \right) \right) \right] t
\]

\[
= \frac{t^2}{2}
\]

(3.93)

Integrating equation (3.93) and applying the initial condition \( c_2(0) = 0 \)

\[
c_2 = \left[ \gamma_H + u_1 \left( \left( \beta_e \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - \left( \mu_H + \mu_0 + \gamma_H + u_1 \right) \right) \right) \right] t^2
\]

\[
= \frac{t^2}{2}
\]

(3.94)

Substituting equation (3.54), (3.72) and (3.94) into equation (3.19) and then take the limit as \( h \to 1 \).
\[ R_H = R_{H(0)} + \left( (\gamma_H + u_i)I_{H(0)} - \mu_H R_{H(0)} \right) t + \]
\[
[\gamma_H + u_i \left( (\beta_\varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_i)I_{H(0)} \right) \\
-\mu_H \left( (\gamma_H + u_i)I_{H(0)} - \mu_H R_{H(0)} \right) ] \frac{t^2}{2} \]

(3.95)

From (3.40)
\[
h^2 : d_1 + e_1 \beta_1 \phi + (\mu_A + u_2) + d_1 = 0 \quad (3.96)
\]
\[
d_2 = (\mu_A + u_2) + d_1 - e_1 \beta_1 \phi \quad (3.97)
\]
Substituting equation (3.76) and (3.79) into equation (3.97),
\[
d_2' = \left[ (\mu_A + u_2) + (\mu_A + u_2 - S_{A(0)} - \beta_1 \epsilon \varphi I_{A(0)} \right) \\
-\left( S_{H(0)} - (\mu_A + u_2)I_{A(0)} + \beta_1 \epsilon \varphi I_{H(0)} \right) \beta_1 \epsilon \varphi \right] t \]

(3.98)
 Integrating equation (3.98) and applying the initial condition \[ d_2(0) = 0 \]
\[
d_2 = \left[ \mu_A + u_2 + (\mu_A + u_2 - S_{A(0)} - \beta_1 \epsilon \varphi I_{A(0)} \right) - \left( S_{H(0)} - (\mu_A + u_2)I_{A(0)} + \beta_1 \epsilon \varphi I_{H(0)} \right) \frac{t^2}{2} \beta_1 \epsilon \varphi \]

(3.99)
Substituting equation (3.57),(3.76) and (3.99) in to equation (3.20)
\[
S_A = S_{A(0)} + (\mu_A + u_2) - \beta_1 \epsilon \varphi I_{A(0)} \right) t + \\
[\mu_A + u_2 + (\mu_A + u_2 - S_{A(0)} - \beta_1 \epsilon \varphi I_{A(0)}) - \\
(\beta_1 \epsilon \varphi I_{H(0)} \right) \beta_1 \epsilon \varphi ] \frac{t^2}{2} \]

(3.100)
From equation (3.45),
\[
h^2 : e_1' + e_1 (\mu_A + u_2) - \beta_1 \epsilon \varphi b_1 + a_1 = 0 \quad (3.101)
\]
\[
e_1' = \beta_1 \epsilon \varphi b_1 + a_1 - e_1 (\mu_A + u_2) \quad (3.102)
\]
Substituting equation (3.68),(3.64) and (3.80) in to equation (3.102)
\[
\left[ \beta_1 \epsilon \varphi \left( \beta_1 \epsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_i)I_{H(0)} \right) \\
+ (\Lambda_H - \beta_1 \epsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H) - \\
(\beta_1 \epsilon \varphi I_{H(0)} \right) \beta_1 \epsilon \varphi I_{H(0)} \mu_A + u_2 \right] t \]

(3.103)

Integrating equation (3.103) and applying the initial condition \[ e_2(0) = 0 \]
NUMERICAL SIMULATIONS OF THE MODEL

With Mapple software we obtained graphical profiles for each compartment using the results obtained above with the following values of variables and parameters.

<table>
<thead>
<tr>
<th>Variables/parameters</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{H}$</td>
<td>1000</td>
<td>Assumed</td>
</tr>
<tr>
<td>$I_{H}$</td>
<td>700</td>
<td>Assumed</td>
</tr>
<tr>
<td>$R_{H}$</td>
<td>500</td>
<td>Assumed</td>
</tr>
<tr>
<td>$I_{A}$</td>
<td>1500</td>
<td>Assumed</td>
</tr>
<tr>
<td>$\Lambda_{H}$</td>
<td>1000/day</td>
<td>Bonya and Okosun(2016)</td>
</tr>
</tbody>
</table>
### Table 4.1: Variable and Parameters Values of the model

<table>
<thead>
<tr>
<th>Variable/parameters</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_A$</td>
<td>100/day</td>
<td>Bonyah and Okosun(2016)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.2/day</td>
<td>Mojumder et al. (2016)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.09</td>
<td>Okosun et al. -2010</td>
</tr>
<tr>
<td>$\mu_A$</td>
<td>14-Jan</td>
<td>Mojumder et al. (2016)</td>
</tr>
<tr>
<td>$\mu_H$</td>
<td>$1/(365.600)/day$</td>
<td>Nishura et al (2016)</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>100</td>
<td>Assumed</td>
</tr>
<tr>
<td>$\gamma_H$</td>
<td>0.01</td>
<td>Mojumder et al. (2016)</td>
</tr>
</tbody>
</table>

### Graphical Profiles of Each Compartment

- **Figure 4.1**: Showing the graph of infected individuals against time for different values of biting and contact rates.
- **Figure 4.2**: Show the graph of infected human against time for different treatment rates.
- **Figure 4.3**: Displays the graph of Recovered human against time for varied treatment rate and the use of insecticide.
- **Figure 4.4**: Shows the graph of infected Aedes Mosquitoes against time for different values of biting and contact rates.
Discussion of Results

Figure 4.1 shows the graph of individual infected with Zika virus infection against time for different values of biting and contact rate. The graph shows that as the biting and contact rate increases the number of infected humans also increases (i.e $\varepsilon = 0.01, \varphi = 0.01, \varepsilon = 0.02, \varphi = 0.02$ and $\varepsilon = 0.03, \varphi = 0.03$). It is observed that the population of Zika virus infected individuals’ increases as the Zika virus contact parameter also increases. With this in mind, the use of physical barriers such as regular or mesh screens or insecticide treated netting materials on doors and windows should be observed, pregnant women to postpone non-critical travel to areas with ongoing Zika virus transmission and travelers from areas with ongoing Zika virus transmission to practice safer sex and not to donate blood for at least one month after return to Nigeria, to reduce the potential risk of onwards transmission.

Figure 4.2 is the graph of infected human against time for different treatment rate. The graph shows that as the control strategy increases the infected human population decreases (i.e $\mu_1 = 0.25, \mu_2 = 0.20, \mu_1 = 0.55, \mu_1 = 0.50, \mu_1 = 0.75, \mu_2 = 0.70$).

Figure 4.3 displays the graph of individuals recovered human from Zika virus infection $R(t)$ against time for varied treatment rate and the use of insecticide. The graph shows that as the control strategy increases, the number of recovered individuals also increases (i.e $\mu_1 = 0.25, \mu_2 = 0.20, \mu_1 = 0.55, \mu_1 = 0.50, \mu_1 = 0.75, \mu_2 = 0.70$). Due to the deadly nature of severe Zika virus infection,
prevention and early treatment of Zika virus should be encouraged to increase the number of recovered individuals from Zika virus infection.

Figure 4.4 shows the graph of infected aedes mosquitoes against time for different values of biting and contact rate. The graph shows that as the biting and contact rate increases the no of infected aedes mosquitoes increases in population. (i.e $\varepsilon = 0.01, \varphi = 0.01$, $\varepsilon = 0.02, \varphi = 0.02$ and $\varepsilon = 0.03, \varphi = 0.03$). We observed that as the biting and contact rate increases the number of aedes mosquitoes increases. It is encouraged that insecticides that cover larger area should be used at faster time so as to stop the vectors from reproducing due to the proximity amongst themselves.

Figure 4.5 shows the graph of infected aedes mosquito against time for varied treatment rate and the use of insecticide (i.e $\mu_1 = 0.25, \mu_2 = 0.20, \mu_1 = 0.55, \mu_1 = 0.50, \mu_1 = 0.75, \mu_2 = 0.70$). It is observed that as the control strategies increases the infected aedes mosquitoes decreases. Therefore it is encouraged to use insecticide and treatment in order to reduce and eradicate the virus completely in Zika endemic areas.

### Conclusion

In this study, the impact of treatment and insecticide on the burden of Zika Virus infection was investigated. Appropriate control strategy for the infection eradication and control, was also examined. The result of the numerical simulation indicates that treatment and the use of insecticides is the best strategy to control the disease.

The model shows that the spread of zika virus infection depends largely on the biting and contact rates; hence the National zika virus control program should emphasize on the improvement in early detection of zika virus cases so that transmission can be minimized.

Infectious individuals should be isolated and treated immediately.

### REFERENCES


