VOL. 15 NO. 9
ISSN: 2278-8881
SEPTEMBER, 2019

African Scholar Publications & Research International

African Scholars Journal of pure and Applied Science (JPAS-9)

Magnetohydrodynamic Casson Fluid Flow over an Exponential Stretching Sheet with Effect of Radiation

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Abstract

Magnetohydrodynamics (MHD) casson fluid flow over an exponentially stretching sheet is investigated with effect of radiation. The governing partial differential equations were reduced to ordinary differential equations using similarity transformation. The reduced non-linear ordinary differential equations were solved using iteration perturbation method and the results obtained were presented graphically. The effects of casson parameter, radiation parameter, magnetic parameter, heat source/sink on the velocity and temperature profiles were discussed. It is revealed that these parameters play crucial role on MHD casson fluid flow.

Keywords: *MHD, Casson fluid, Stretching sheet.*

Introduction

Due to the increasing importance of non – Newtonian fluids in industry, the stretching sheet concept has recently extended to fluids obeying non- Newtonian consecutive equation Kumar and Gangadhar (2015). Casson fluid is one type of non – Newtonian fluid, it can be defined as a shear tinning liquid which is supposed to have an infinite viscosity at zero rate of shear and a yield stress under which no flow occurs and zero viscosity at an infinite rate of shear (Sharada and Shnkar, 2016).

The viscosity of a non-Newtonian fluid will change due to agitation or pressure technically known as shear stress. A fluid in which the viscous stress arising from its flow at every point are linearly proportional to the rate of change of its deformation over time is called Newtonian fluid Emmanuel *et al.* (2015). Fluids for which the shear stress is not linearly related to the shear strain rate are called non-Newtonian fluids. Examples include slurries and colloidal suspensions, polymer solutions, blood, paste, and cake batter. Some non-Newtonian fluids exhibit a "memory"-the shear stress depends not only on local strain rate, but also on its history.

Kushpalalata *et al*. (2017) analyzed the effects of cross diffusion on casson fluid over an unsteady stretching surface with boundary effects. The governing equations were solved numerically using Runge-Kutta fourth order along with shooting technique.

Maleque (2016) investigated an exothermic/endothermic binary chemical reaction on unsteady MHD non-Newtonian casson fluid flow with heat and mass transfer past a flat porous plate. Considering the effects of casson parameter on the velocity profile for cooling and heating plate, the exothermic/endothermic chemical reaction rate and Arrhenius energy on the concentration. The governing equations were solved numerically by adopting implicit Runge-Kutta and shooting method using the Nachtsheim-Swigert iteration technique. Maleque (2013) investigated the effects of exothermic/endothermic chemical reaction with Arrhenius activation energy on MHD free convection mass transfer flow in presence of thermal radiation. The governing equations were solved numerically by adopting Runge-Kutta and shooting method using the Nachtsheim-Swigert iteration technique. Prakash *et al.* (2016) examined the thermal and solutal boundary layer in incompressible, laminar flow over an exponentially stretching sheet with variable temperature and concentration in the presence of chemical reaction and thermal radiation. The governing partial differential equations were transformed into self-similar ordinary differential equation and solved by using Matlab bvp4c package. Charankumar *et al.* (2016) examined chemical reaction and Soret effects on casson MHD fluid flow over a vertical plate with heat source/sink. The problem was solved numerically using perturbation technique for the velocity, the temperature and the concentration species. Kumar and Gangadhar (2015) investigated the interactions of MHD stagnation point of electrically conducting non-Newtonian casson fluid and heat Transfer towards a stretching sheet in the presence of viscous dissipation, momentum and thermal slip flow. The governing equations were transformed into self-similar nonlinear Ordinary differential equations and solved numerically.

Mathematical Formulation

We consider three dimensional (3D) steady incompressible flows over an exponentially stretching sheet. The sheet is stretched along the xy plane, while the fluid is placed along the z - axis; the sheet $z=0$, the uniform magnetic field is applied in z - direction that is perpendicular to the flow direction. Here, we assumed that the sheet was stretched with velocities $U_w = U_0 e^{\frac{x+y}{L}}$ and $V_w = V_0 e^{\frac{x+y}{L}}$ along the xy -plane respectively, where U_0 and V_0 are constants. A heat source/sink placed within the flow to allow for heat generation or absorption effects.

The rheological equation of state for an isotropic flow of casson fluid as stated by Kumar and Gangadhar (2015). can be expressed as

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{p_z}{\sqrt{2\pi}}\right)e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + \dfrac{p_z}{\sqrt{2\pi_c}}\right)e_{ij}, \pi < \pi_c \end{cases} \quad (1)$$

In the above equation $\pi = e_{ij}e_{ij}$ and e_{ij} denotes the $(i, j)^{th}$ components of the deformation rate, π is the product of the deformation rate itself, π_c is the critical value of this product based on the non-Newtonian fluid model, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid and p_z is the yield stress of the fluid. From (3.1) we obtain $\mu_B = \frac{1}{2}\frac{\tau_{ij}}{e_{ij}} - \frac{p_z}{\sqrt{2\pi}}$, $\upsilon = \frac{\mu_B}{\rho}$ and $\beta = \frac{\sqrt{2\pi_c}}{p_z}\mu_B$

The governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\upsilon\left(1+\frac{1}{\beta}\right)\left[\frac{\partial^2 u}{\partial z^2}\right]-\frac{\sigma B^2}{\rho}u+g\beta_T\left(T-T_\infty\right) \quad (3)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\upsilon\left(1+\frac{1}{\beta}\right)\left[\frac{\partial^2 v}{\partial z^2}\right]-\frac{\sigma B^2}{\rho}v \quad (4)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\left[\frac{\partial^2 T}{\partial z^2}\right]+\frac{Q_1}{\rho c_p}\left(T-T_\infty\right)-\frac{\partial q_r}{\partial z} \quad (5)$$

The radiatve heat term, using the Roseland approximation as in Kumar and Gangadhar (2015) is given by, $q_r=-\frac{4\sigma_1}{3k_1}\frac{\partial\left(T^4\right)}{\partial z}$ (6)

The initial and boundary conditions for the problem are given by

$$\left.\begin{array}{l} u=U_w, v=V_w, T=T_w, at\ z=0 \\ u\to 0, v\to 0, T\to T_\infty,\ at\ z\to\infty \end{array}\right\} \quad (7)$$

Where

$U_w=U_0 e^{\frac{x+y}{L}}, V_w=V_0 e^{\frac{x+y}{L}}, T_w=T_\infty+T_0 e^{\frac{x+y}{L}}$,

u, v and w are the velocity component in the direction of x, y and z respectively, β is the casson fluid parameter, υ is the kinematic viscosity, B is the magnetic induction, B_0 is constant, T is temperature of the fluid, β_T is the coefficient of volume expansion for temperature differences, β_{T_0} is constants, Q_1 is heat generation term ($Q_1>0$) or absorption ($Q_1<0$) coefficient, Q_0 is a constant, k thermal diffusivity , δ is the density of the fluid, g_g is acceleration due to gravity, σ is the electrical conductivity, c_p is the specific heat capacity at constant pressure, T_∞ is the free stream temperature, k is the Boltzmann constant k_0 is constant and the radiative heat flux q_r is described by Roseland approximation such that $q_r=-\frac{4\sigma_1}{3k_1}\frac{\partial\left(T^4\right)}{\partial z}$ where σ_1 and k_1 are the Stefan Boltzmann constant and mean absorption coefficient respectively.

Method of Solution

The governing partial differential equations were reduced to ordinary differential equations using similarity transformation. The reduced non-linear ordinary differential equations were solved using iteration perturbation method as used in Mohammed *et al*. (2015).

Introducing similarity parameters:

$$\left.\begin{aligned}&\eta=\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x+y}{2L}}z, u=U_0e^{\frac{x+y}{L}}f'(\eta), v=U_0e^{\frac{x+y}{L}}g'(\eta), T=T_\infty+T_0e^{\frac{x+y}{L}}\theta(\eta),\\&w=-\sqrt{\frac{\upsilon U_0}{2L}}e^{\frac{x+y}{2L}}\left(f+\eta f'+g+\eta g'\right), \beta_T=\beta_{T_0}e^{\frac{x+y}{L}}, B=B_0e^{\frac{x+y}{2L}}, Q=Q_0e^{\frac{x+y}{L}}\end{aligned}\right\} \quad (6)$$

The Transformed Equations together with the initial and boundary conditions are:

$$c_1f'''(\eta)=2f'(\eta)\left(f'+\frac{\eta}{2}f''\right)+2g'(\eta)\left(f'+\frac{\eta}{2}f''\right)-\left(f+\eta f'+g+\eta g'\right)f''(\eta)$$
$$+Mf'(\eta)-G_{r_\theta}\theta(\eta) \quad (9)$$

$$c_1g'''(\eta)=2f'(\eta)\left(g'+\frac{\eta}{2}g''\right)+2g'(\eta)\left(g'+\frac{\eta}{2}g''\right)-\left(f+\eta f'+g+\eta g'\right)g''(\eta)+Mg'(\eta) \quad (10)$$

$$c_2\theta''=2f'(\eta)\left(\theta(\eta)+\frac{\eta}{2}\theta'(\eta)\right)+2g'(\eta)\left(\theta(\eta)+\frac{\eta}{2}\theta'(\eta)\right)-\left(f+\eta f'+g+\eta g'\right)\theta'(\eta)-Q\theta(\eta)(\eta) \quad (11)$$

$$\left.\begin{aligned}&f(0)=0, \quad g(0)=0, \; f'(0)=1, \quad g'(0)=c, \quad \theta(0)=1,\\&f'\to 0 \;\; as \;\; \eta\to\infty, \;\; g'\to 0 \;\; as \;\; \eta\to\infty, \;\; \theta\to 0 \;\; as \;\; \eta\to\infty,\end{aligned}\right\} \quad (12)$$

We rewrite equations (9) – (12) in the form:

$$c_1f'''+bf''+\left(f+\eta f'+g+\eta g'-b\right)f''-2\left(f'+g'\right)\left(f'+\frac{\eta}{2}f''\right)-Mf'+G_{r_\theta}\theta=0 \quad (13)$$

$$c_1g'''+bg''+\left(f+\eta f'+g+\eta g'-b\right)g''-2\left(f'+g'\right)\left(g'+\frac{\eta}{2}g''\right)-Mg'=0 \quad (14)$$

$$c_2\theta''+b\theta'+\left(f+\eta f'+g+\eta g'-b\right)\theta'-2\left(f'+g'\right)\left(\theta+\frac{\eta}{2}\theta'\right)+Q\theta=0 \quad (15)$$

$$
\left.\begin{array}{l} f(0)=0, \quad g(0)=0, \; f'(0)=1, \quad g'(0)=c, \quad \theta(0)=1, \\ f'\to 0 \;\; as \;\; \eta\to\infty, \;\; g'\to 0 \;\; as \;\; \eta\to\infty, \;\; \theta\to 0 \;\; as \;\; \eta\to\infty, \end{array}\right\} \tag{16}
$$

Using the initial approximation:

$$
\left.\begin{array}{l} f_0(\eta)=\dfrac{1}{b}\left(1-e^{-b\eta}\right) \\ g_0(\eta)=\dfrac{c}{b}\left(1-e^{-b\eta}\right) \end{array}\right\} \tag{17}
$$

Substituting (17) into (13)-(15), we obtain:

$$
c_1 f''' + bf'' + \left(\frac{1}{b}\left(1-e^{-b\eta}\right)+\eta f' + \frac{c}{b}\left(1-e^{-b\eta}\right)+\eta g' - b\right)f'' - 2\left(f'+g'\right)\left(f'+\frac{\eta}{2}f''\right) - Mf' + G_{r_\theta}\theta = 0 \tag{18}
$$

$$
c_1 g''' + bg'' + \left(\frac{1}{b}\left(1-e^{-b\eta}\right)+\eta f' + \frac{c}{b}\left(1-e^{-b\eta}\right)+\eta g' - b\right)g'' - 2\left(f'+g'\right)\left(g'+\frac{\eta}{2}g''\right) - Mg' = 0 \tag{19}
$$

$$
c_2\theta'' + b\theta' + \left(\frac{1}{b}\left(1-e^{-b\eta}\right)+\eta f' + \frac{c}{b}\left(1-e^{-b\eta}\right)+\eta g' - b\right)\theta' - 2\left(f'+g'\right)\left(\theta+\frac{\eta}{2}\theta'\right) + Q\theta = 0 \tag{20}
$$

Embedding an artificial parameter ε into (43)-(45), we have:

$$
c_1 f''' + bf'' + \varepsilon\left(\frac{1}{b}\left(1-e^{-b\eta}\right)+\eta f' + \frac{c}{b}\left(1-e^{-b\eta}\right)+\eta g' - b\right)f'' - 2\varepsilon\left(f'+g'\right)\left(f'+\frac{\eta}{2}f''\right) - \varepsilon Mf' + \varepsilon G_{r_\theta}\theta = 0 \tag{21}
$$

$$
c_1 g''' + bg'' + \varepsilon\left(\frac{1}{b}\left(1-e^{-b\eta}\right)+\eta f' + \frac{c}{b}\left(1-e^{-b\eta}\right)+\eta g' - b\right)g'' - 2\varepsilon\left(f'+g'\right)\left(g'+\frac{\eta}{2}g''\right) - \varepsilon Mg' = 0 \tag{22}
$$

$$
c_2\theta'' + b\theta' + \varepsilon\left(\frac{1}{b}\left(1-e^{-b\eta}\right)+\eta f' + \frac{c}{b}\left(1-e^{-b\eta}\right)+\eta g' - b\right)\theta' - 2\varepsilon\left(f'+g'\right)\left(\theta+\frac{\eta}{2}\theta'\right) + \varepsilon Q\theta = 0 \tag{23}
$$

Expressing the solution of (21)-(23), in the form:

$$
\begin{array}{l} f(\eta) = f_0(\eta) + \varepsilon f_1(\eta) + \ldots \\ g(\eta) = g_0(\eta) + \varepsilon g_1(\eta) + \ldots \\ \theta(\eta) = \theta_0(\eta) + \varepsilon\theta_1(\eta) + \ldots \end{array} \tag{24}
$$

We have:

$$
c_1 f_0''' + bf_0'' = 0, \quad f_0(0)=0, \quad f_0'(0)=1, \quad f_0'(\infty)=0 \tag{25}
$$

$$
c_1 g_0''' + bg_0'' = 0, \quad g_0(0)=0, \quad g_0'(0)=c, \quad g_0'(\infty)=0 \tag{26}
$$

$$
c_2\theta_0'' + b\theta_0' = 0, \quad \theta_0(0)=1, \quad \theta_0(\infty)=0 \tag{27}
$$

$$c_1 f_1''' + b f_1'' + \left(\frac{1}{b}\left(1-e^{-b\eta}\right) + \eta f_0' + \frac{c}{b}\left(1-e^{-b\eta}\right) + \eta g_0' - b\right) f_0'' - 2\left(f_0' + g_0'\right)\left(f_0' + \frac{\eta}{2} f_0''\right)$$
$$- M f_0' + G_{r_\theta}\theta_0 = 0,\ f_1(0)=0, \quad f_1'(0)=1, \quad f_1'(\infty)=0 \tag{28}$$

$$c_1 g_1''' + b g_1'' + \left(\frac{1}{b}\left(1-e^{-b\eta}\right) + \eta f_0' + \frac{c}{b}\left(1-e^{-b\eta}\right) + \eta g_0' - b\right) g_0'' - 2\left(f_0' + g_0'\right)\left(g_0' + \frac{\eta}{2} g_0''\right)$$
$$- M g_0' = 0, \quad g_1(0)=0, \quad g_1'(0)=c, \quad g_1'(\infty)=0 \tag{29}$$

$$c_2\theta_1'' + b\theta_1' + \left(\frac{1}{b}\left(1-e^{-b\eta}\right) + \eta f_0' + \frac{c}{b}\left(1-e^{-b\eta}\right) + \eta g_0' - b\right)\theta_0' - 2\left(f_0' + g_0'\right)\left(\theta_0 + \frac{\eta}{2}\theta_0'\right) + Q\theta_0 = 0$$
$$\theta_1(0)=1, \quad \theta_1(\infty)=0 \tag{30}$$

Solution of equations (25) – (30) gives:

$$f(\eta) = \frac{1}{p_1}\left(1-e^{-p_1\eta}\right) + \varepsilon\left(p_5 e^{-p_{10}\eta} - p_6\eta e^{-p_1\eta} - 2p_7 e^{-p_1\eta} - p_8 e^{-p_2\eta} - p_9 e^{-2p_1\eta} + \frac{p_3}{p_1^2} e^{-p_1\eta} + p_4\right) \tag{31}$$

$$g(\eta) = \frac{c}{p_1}\left(1-e^{-p_1\eta}\right) + \varepsilon\left(p_{13} e^{-p_{10}\eta} - p_{14}\eta e^{-p_1\eta} - 2p_{15} e^{-p_1\eta} - p_{16} e^{-2p_1\eta} + \frac{p_{11}}{p_1^2} e^{-p_1\eta} + p_{12}\right) \tag{32}$$

$$\theta(\eta) = e^{-p_2\eta} + \varepsilon\left(p_{18}\eta e^{-p_2\eta} + p_{19} e^{-p_2\eta} - p_{20} e^{-p_{22}\eta} + p_{21} e^{-p_{23}\eta} - \frac{p_{17}}{p_2} e^{-p_2\eta}\right) \tag{33}$$

Where,

$$p_1 = \frac{b}{c_1},\ p_2 = \frac{b}{c_2},\ \ p_3 = \left(\frac{1}{p_1^2 c_1}\left(bp_1 - \frac{p_1}{b} - \frac{p_1 c}{b} - M\right) - \frac{1}{bc_1(p_1+b)}\left(\frac{p_1}{b} + \frac{p_1 c}{b}\right) + \frac{1}{2c_1 p_1^2}(2+2c) + \frac{G_{r_\theta}}{p_2 c_1(p_1-p_2)} - 1\right)p_1,$$

$$p_4 = 2\frac{1}{p_1^3 c_1}\left(bp_1 - \frac{p_1}{b} - \frac{p_1 c}{b} - M\right) - \frac{1}{bc_1(p_1+b)^2}\left(\frac{p_1}{b} + \frac{p_1 c}{b}\right) + \frac{1}{4c_1 p_1^3}(2+2c) + \frac{G_{r_\theta}}{p_2^2 c_1(p_1-p_2)} - \frac{p_3}{p_1^2};$$

$$p_5 = \frac{1}{bc_1(p_1+b)^2}\left(\frac{p_1}{b} + \frac{p_1 c}{b}\right),\ p_6 = \frac{1}{p_1^2 c_1}\left(bp_1 - \frac{p_1}{b} - \frac{p_1 c}{b} - M\right),\ p_7 = \frac{1}{p_1^3 c_1}\left(bp_1 - \frac{p_1}{b} - \frac{p_1 c}{b} - M\right),$$

$$p_8 = \frac{G_{r_\theta}}{p_2^2 c_1(p_1-p_2)},\ p_9 = \frac{1}{4c_1 p_1^3}(2+2c),\ p_{10} = p_1 + b,$$

$$p_{11}=\left(\frac{1}{c_1p_1^2}\left(bcp_1-\frac{p_1c}{b}-\frac{p_1c^2}{b}-Mc\right)-\frac{1}{bc_1(p_1+b)}\left(\frac{p_1c}{b}+\frac{p_1c^2}{b}\right)+\frac{1}{c_1p_1}\left(c+c^2\right)-c\right)p_1$$

$$p_{12}=\frac{1}{2c_1p_1}\left(c+c^2\right)-\frac{1}{bc_1(p_1+b)^2}\left(\frac{p_1c}{b}+\frac{p_1c^2}{b}\right)+2\frac{1}{c_1p_1^3}\left(bcp_1-\frac{p_1c}{b}-\frac{p_1c^2}{b}-Mc\right)+\frac{p_{11}}{p_1^2}$$

$$p_{13}=\frac{1}{bc_1(p_1+b)^2}\left(\frac{p_1c}{b}+\frac{p_1c^2}{b}\right),\ p_{14}=\frac{1}{c_1p_1^2}\left(bcp_1-\frac{p_1c}{b}-\frac{p_1c^2}{b}-Mc\right),$$

$$p_{15}=\frac{1}{c_1p_1^3}\left(bcp_1-\frac{p_1c}{b}-\frac{p_1c^2}{b}-Mc\right),\ p_{16}=\frac{1}{2c_1p_1}\left(c+c^2\right)$$

$$p_{17}=\left(\frac{1}{c_2p_2^2}\left(p_2b-\frac{p_2}{b}-\frac{p_2c}{b}+Q\right)-\frac{1}{bc_2(b+p_2)}\left(\frac{p_2}{b}+\frac{p_2c}{b}\right)+\frac{1}{c_2p_1(p_1+p_2)}(2+2c)-1\right)p_2,$$

$$p_{18}=\frac{1}{c_2p_2}\left(p_2b-\frac{p_2}{b}-\frac{p_2c}{b}+Q\right),\ p_{19}=\frac{1}{c_2p_2^2}\left(p_2b-\frac{p_2}{b}-\frac{p_2c}{b}+Q\right),\ p_{20}=\frac{1}{bc_2(b+p_2)}\left(\frac{p_2}{b}+\frac{p_2c}{b}\right),$$

$$p_{21}=\frac{1}{c_2p_1(p_1+p_2)}(2+2c),\ p_{22}=b+p_2,\ p_{23}=p_1+p_2$$

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