# QUALITATIVE BEHAVIOUR OF THE SOLUTION TRAJECTORIES OF REAL TIME COMPETITIVE CONTINUOUS INTERACTING SYSTEMS: ISSUES FOR INITIAL DATA, DURATION OF GROWTH AND STABILIZATION

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**ABSTRACT**
*The model equations describing the interaction between two competing yeast species have been fully stabilized using the technique of a numerical simulation undergoing changing length of the growing season and the initial data. The results of this present study have not been seen elsewhere; these are presented and discussed in this paper.*

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## Introduction

One of the methods of studying the qualitative behaviour of the solution trajectories of real time competitive continuous interacting systems concerns the use of a numerical simulation analysis in terms of the initial data, the duration of growth and stabilization (Ekaka-a, 2009; Ekaka-a, Nafo and Lale, 2014; Ekaka-a, 2009). Other related works can be found in (Kot, 2001; Pielou, 1977; Murray, 1993; Renshaw, 1991; Damgaard, 2004).
The above proposed idea is yet to be extended to tackle other important mathematical equations in the crop science phenomena. One of these crop science phenomena is one of the oldest types of biological interaction hereby called the competitive interaction between two interacting yeast species for limited growth resources. The three factors of the initial data, the length of the growing season and stabilization play significant roles in the effective numerical determination of the convergence behaviour of the solution trajectories to the expected co-existence steady-state solution. On the basis of this conjecture, the correct convergence to the co-existence steady-state solution depends on the initial data and the length of the growing season in the main of which the stable co-existence steady-state solution can be further stabilized. These numerically derived information can provide vital monitoring data for the ecosystem functioning and stability of two interacting seasonally dependent legumes in terms of their joint contribution for the survival of these subsistence crops that have strong bearing on food production and sustainable development.

## Mathematical Formulation

Following Ekaka-a, Nafo and Lale (2014), we have considered the following system of continuous nonlinear first order ordinary differential equations that describes the dynamics between two interacting species of yeast in the perspective of a competition interaction

$$\frac{dC(t)}{dt} = C(t)[\ _1 - \ _1 C(t) - \ _1 G(t)]$$

(1)

$$\frac{dG(t)}{dt} = G(t)[\ _2 - \ _2 G(t) - \ _2 C(t)]$$

(2)

Here, the model parameter values  and specify the intrinsic growth rates having the precise values of 0.1 and 0.08 in the units of grams over the permitted area of land space; the model parameter values and specify the intra-competition coefficients having the precise values of 0.0014 and 0.001; the model parameter values and specify the inter-competition coefficients having the precise values of 0.0012 and 0.0009. The independent variable t is specified as the unit of days.

**Method of Solution**

In this study, the factors of the length of the growing season and the initial data were varied and utilized to measure the qualitative behaviour of the co-existence steady-state solution for two competing species of yeast. In each typical empirical example, the initial data are assumed to have fixed un-changing values. The method of a numerical simulation has been utilized to study the convergence of the co-existence steady-state solution. The results of exploring this numerical method are presented and discussed next.

**Results and Discussions**

The application of the above defined method of solution has been implemented to produce the following results that are presented and discussed as follows:

Table 1: Testing for the qualitative behaviour of the solution trajectories of real time competitive interacting systems for the initial data (4, 10) when the duration of growth is 10 days

| Example | Duration of growth in days | Initial data | Limiting Converging Values | |
|---|---|---|---|---|
| 1 | 10 | 4,10 | 4.000000000000000 | 10.000000000000000 |
| 2 | 10 | 4,10 | 4.340775940698729 | 10.681311902090316 |
| 3 | 10 | 4,10 | 4.704316430668463 | 11.397461179002367 |
| 4 | 10 | 4,10 | 5.091142463328402 | 12.148604616537666 |
| 5 | 10 | 4,10 | 5.501625618929798 | 12.934661188834772 |
| 6 | 10 | 4,10 | 5.935964655819314 | 13.755288700283659 |
| 7 | 10 | 4,10 | 6.394162735666852 | 14.609862720812226 |
| 8 | 10 | 4,10 | 6.876006096994366 | 15.497458732746789 |
| 9 | 10 | 4,10 | 7.381045070566770 | 16.416838429908708 |
| 10 | 10 | 4,10 | 7.908578375483942 | 17.366441088617989 |

In these ten (10) empirical examples, when the length of the growing season is 10 days and the initial data are 4 and 10 in the appropriate unit, the final converging values are 7.908578375483942 and 17.366441088617989. Hence, when the biomass of yeast species 1 is approximately 7.9 grams per area of growing domain and the biomass of yeast species 2 is approximately 17.4 grams per area of growing domain, the stabilization of the co-existence steady-state solution is yet to be realised. In contrast, when the initial data are 4 and 10 and when the length of the growing season is 4500 days, the co-existence steady-state solution (12.5, 68.75) is approximately stabilized (Table 2).

Table 2: Testing for the qualitative behaviour of the solution trajectories of a real time competitive interacting systems for the initial data (4, 10) when the duration of growth ranges from 200 days to 4600 days

| Example | Duration of growth in days | Initial data | Limiting Converging Values |
|---|---|---|---|
| 11 | 200 | 4,10 | 19.892249567321407 61.544230077467837 |
| 12 | 400 | 4,10 | 15.678465734416708 65.689404187694336 |
| 13 | 1000 | 4,10 | 12.872658152299001 68.392434356157636 |
| 14 | 2000 | 4,10 | 12.503294638148681 68.694011678627334 |
| 15 | 3000 | 4,10 | 12.491830577321746 68.711664047418978 |
| 16 | 4000 | 4,10 | 12.499528889502713 68.747767404134933 |
| 17 | 4300 | 4,10 | 12.499426286284278 68.747381522094841 |
| 18 | 4400 | 4,10 | 12.493748655583241 68.722648686297873 |
| 19 | 4500 | 4,10 | 12.499706034221260 68.748620108620798 |
| 20 | 4600 | 4,10 | 12.494172342121640 68.724439118095276 |

Similarly, Table 3 shows that when the biomass of yeast species 1 is approximately 9.7 grams per area of growing domain and when the biomass of species 2 is approximately 17.2 grams per area of growing domain, the stabilization of the co-existence steady-state solution is yet to be realised. In contrast, when the initial data are 5 and 10 and when the length of the growing season is 4500 days, the co-existence steady-state solution (12.5, 68.75) is approximately stabilized (Table 4).

Table 3: Testing for the qualitative behaviour of the solution trajectories of a real time competitive interacting systems for the initial data (5, 10) when the duration of growth is 10 days

| Example | Duration of growth in days | Initial data | Limiting Converging Values |
|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| 21 | 10 | 5,10 | 5.000000000000000 | 10.000000000000000 |
| 22 | 10 | 5,10 | 5.418119229049101 | 10.671386008562497 |
| 23 | 10 | 5,10 | 5.862817624002136 | 11.375600848882785 |
| 24 | 10 | 5,10 | 6.334467401958969 | 12.112556928495174 |
| 25 | 10 | 5,10 | 6.833233872556615 | 12.881918841332064 |
| 26 | 10 | 5,10 | 7.359049102645878 | 13.683083615455537 |
| 27 | 10 | 5,10 | 7.911587513109406 | 14.515164116014668 |
| 28 | 10 | 5,10 | 8.490244489596373 | 15.376976510283198 |
| 29 | 10 | 5,10 | 9.094119134081549 | 16.267032676441449 |
| 30 | 10 | 5,10 | 9.722002273436500 | 17.183538362437265 |

Table 4: Testing for the qualitative behaviour of the solution trajectories of a real time competitive interacting systems for the initial data (5, 10) when the duration of growth ranges from 200 days to 4600 days

| Example | Duration of growth in days | Initial data | Limiting Converging Values | |
|---|---|---|---|---|
| 31 | 200 | 5,10 | 21.120312470574035 | 60.284720700503073 |
| 32 | 400 | 5,10 | 16.132453504364005 | 65.278383650202144 |
| 33 | 1000 | 5,10 | 12.925136777377585 | 68.383509975746790 |
| 34 | 2000 | 5,10 | 12.508086873571179 | 68.706701042461603 |
| 35 | 3000 | 5,10 | 12.499678270611478 | 68.745657990958236 |
| 36 | 4000 | 5,10 | 12.501666979789341 | 68.757078597167293 |
| 37 | 4300 | 5,10 | 12.499310801598551 | 68.746867036402961 |
| 38 | 4400 | 5,10 | 12.490165759160046 | 68.707050035595984 |
| 39 | 4500 | 5,10 | 12.499450532803561 | 68.747498870210208 |
| 40 | 4600 | 5,10 | 12.498387684928712 | 68.742866744948813 |

Similarly, Table 5 shows that when the biomass of yeast species 1 is approximately 11.5 grams per area of growing domain and when the biomass of yeast species 2 is approximately 17.0 grams per area of growing domain, the stabilization of the co-existence steady-state solution is yet to be realised. In contrast, when the initial data are 6 and 10 and when the length of the growing season is 4500 days, the co-existence steady-state solution (12.5, 68.75) is approximately stabilized (Table 6).

Table 5: Testing for the qualitative behaviour of the solution trajectories of a real time competitive interacting systems for the initial data (6, 10) when the duration of growth is 10 days

| Example | Duration of growth in days | Initial data | Limiting Converging Values |
|---|---|---|---|

| | | | |
|---|---|---|---|
| 41 | 10 | 6,10 | 6.000000000000000 10.000000000000000 |
| 42 | 10 | 6,10 | 6.492349473692558 10.661483682053465 |
| 43 | 10 | 6,10 | 7.014413504765979 11.353847619502526 |
| 44 | 10 | 6,10 | 7.566334367218672 12.076782393822159 |
| 45 | 10 | 6,10 | 8.147989444980100 12.829725564048589 |
| 46 | 10 | 6,10 | 8.758964103922935 13.611845944169222 |
| 47 | 10 | 6,10 | 9.398528141085032 14.422031785258174 |
| 48 | 10 | 6,10 | 10.065617104456077 15.258883709972997 |
| 49 | 10 | 6,10 | 10.758819756509045 16.120713172247914 |
| 50 | 10 | 6,10 | 11.476372856741744 17.005547087181778 |

Table 6: Testing for the qualitative behaviour of the solution trajectories of a real time competitive interacting systems for the initial data (6, 10) when the duration of growth ranges from 200 days to 4600 days

| Example | Duration of growth in days | Initial data | Limiting Converging Values |
|---|---|---|---|
| 51 | 200 | 6,10 | 22.115332487633715 59.267532968860372 |
| 52 | 400 | 6,10 | 16.481793705251004 64.937067910680256 |
| 53 | 1000 | 6,10 | 12.959872429994116 68.355614628001561 |
| 54 | 2000 | 6,10 | 12.510732054000941 68.712058187059725 |
| 55 | 3000 | 6,10 | 12.500151151040948 68.747496893442346 |
| 56 | 4000 | 6,10 | 12.501909034756014 68.758126019931325 |
| 57 | 4300 | 6,10 | 12.499040712082296 68.745685697621070 |
| 58 | 4400 | 6,10 | 12.491349749953983 68.712214159891460 |
| 59 | 4500 | 6,10 | 12.498970549171137 68.745404616524667 |
| 60 | 4600 | 6,10 | 12.498902244824345 68.745110397903417 |

This present contribution to knowledge is an extension of our previous analysis (Ekaka-a, Nafo and Lale, 2014) that reinforces the fact that the expected co-existence steady-state solution can be fully stabilized for different values of the initial data at the same length of the growing season of 4500 days. Mathematically, as the duration of growth approaches 4500 days, the stable co-existence steady-state solution (12.5, 68.75) having two negative eigenvalues -0.0033 and -0.0829 is said to be fully further stabilized. Qualitatively, in the longer period of time, the two negative eigenvalues will contribute to the decaying behaviour of the solution trajectories of the two interacting populations.

**Conclusion**

We have successfully implemented the technique of a numerical simulation to stabilize a stable co-existence steady-state solution further for more than one initial data due to the changing values of the length of the growing season in the context of two competing yeast species for limited growth resources in a deterministic sense.

**References**


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