NUMERICAL SIMULATION ANALYSIS AND STABILIZATION OF TWO INTERACTING MUTUALISTIC CONTINUOUS INTERACTING SYSTEMS UNDERGOING CHANGING INITIAL DATA AND DURATION OF GROWTH

E. N. EKAKA-A
Department of Mathematics and Computer Science, Rivers State University of Science and Technology, Port Harcourt, Nigeria

ABSTRACT
The model equations that describe the mutualistic interaction between two yeast species can be stabilized using the technique of a numerical simulation on the premise of the changing length of the growing season and the initial data. The results of this present study have not been seen elsewhere; these are presented and discussed in this paper.

Introduction

In the context of two interacting mutualistic populations, one of the methods of studying the qualitative behaviour of the solution trajectories of real time these continuous interacting systems concerns the use of a numerical simulation analysis in terms of the initial data, the duration of growth and stabilization (Ekaka-a, 2009; Ekaka-a, Nafo and Lale, 2014; Ekaka-a, 2009). Other related works can be found in (Kot, 2001; Pielou, 1977; Murray, 1993; Renshaw, 1991; Damgaard, 2004).

Mathematical Formulation

Following Ekaka-a, Nafo and Lale (2014), we have considered the following system of continuous nonlinear first order ordinary differential equations that describes the dynamics between two interacting species of yeast in the perspective of a mutualistic interaction

$$\frac{dC(t)}{dt} = C(t)[\ _1 - \ _1 C(t) + \ _1 G(t)]$$

(1)

$$\frac{dG(t)}{dt} = G(t)[\ _2 - \ _2 G(t) + \ _2 C(t)]$$

(2)

Here, the model parameter values and specify the intrinsic growth rates having the precise values of 0.1 and 0.1 in the units of grams over the permitted area of land space; the model parameter values and specify the intra-competition coefficients having the precise values of 0.0014 and 0.001; the model parameter values and specify the intercompetition coefficients having the precise values of 0.0012 and 0.0009. The independent variable t is specified as the unit of days.

Method of Solution

In this study, the factors of the length of the growing season and the initial data were varied and utilized to measure the qualitative behaviour of the co-existence steady-state solution for two species of yeast that are engaged in a mutualistic interaction. In each typical empirical example, the initial data are assumed to have fixed un-changing values. The method of a numerical simulation has been utilized to study the convergence of the co-existence steady-state solution. The results of exploring this numerical method are presented and discussed next.

Results and Discussions

The application of the above defined method of solution has been implemented to produce the following results that are presented and discussed as follows:

Table 1: Testing for the qualitative behaviour of the solution trajectories of real time mutualistic interacting systems for the initial data (4, 10) when the duration of growth is 10 days

Example	Duration of growth in days	Initial data	Limiting Converging Values
1	10	4,10	4.000000000000000 10.000000000000000
2	10	4,10	4.450263818906428 10.978095784810655
3	10	4,10	4.953981160216942 12.044714116102345
4	10	4,10	5.517967236560211 13.206591433946787
5	10	4,10	6.149964067698633 14.470778684666884
6	10	4,10	6.858766033344645 15.844634270749721
7	10	4,10	7.654358780769002 17.335818258997648
8	10	4,10	8.548071053293349 18.952290243455234
9	10	4,10	9.552738053489389 20.702314038903779
10	10	4,10	10.682873616470948 22.594473225106832

In these ten (10) empirical examples, when the length of the growing season is 10 days and the initial data are 4 and 10 in the appropriate unit, the final converging values are . 10.682873616470948 and 22.594473225106832. Hence, when the biomass of yeast species 1 is approximately 10.7 grams per area of the growing domain and the biomass of yeast species 2 is approximately 20.6 grams per area of the growing domain, the stabilization of the co-existence steady-state solution is yet to be realised. In contrast, when the initial data are 4 and 10 and when the length of the growing season is 4500 days, the co-existence steady-state solution (687.55, 718.73) is approximately stabilized (Table 2).

Table 2: Testing for the qualitative behaviour of the solution trajectories of real time mutualistic interacting systems for the initial data (4, 10) when the duration of growth ranges from 200 days to 4800 days

Example	Duration of growth in days	Initial data	Limiting Converging Values (times 100)
11	200	4,10	6.870752469503181 7.190701232003380
12	400	4,10	6.875426401919300 7.187249349991213
13	1000	4,10	6.874892744568652 7.187646904488605
14	2000	4,10	6.878692927686709 7.184847854969952
15	3000	4,10	6.874526512369630 7.187921927954868
16	4000	4,10	6.875772633588793 7.187018167767380
17	4300	4,10	6.873235624831872 7.189019837214005
18	4400	4,10	6.873936956877047 7.188489386228095
19	4500	4,10	6.875541237697085 7.187268188286275
20	4600	4,10	6.876414607696531 7.186603014258561
21	4700	4,10	6.874208056935105 7.188238156498744
22	4800	4,10	6.881465927700265 7.182916098526396

Similarly, Table 3 shows that when the biomass of yeast species 1 is approximately 687.38 grams per area of the growing domain and when the biomass of species 2 is approximately 718.86 grams per area of the growing domain, the stabilization of the co-existence steady-state solution is yet to be realised. In contrast, when the initial data are 6 and 10 and when the length of the growing season is 4800 days, the co-existence steady-state solution (687.26, 718.95) is approximately stabilized (Table 4).

Table 3: Testing for the qualitative behaviour of the solution trajectories of real time mutualistic interacting systems for the initial data (5, 10) when the duration of growth ranges from 200 days to 4800 days

Example	Duration of growth in days	Initial data	Limiting Converging Values (times 100)

23	200	5,10	6.871015485747024 7.190787907961136
24	400	5,10	6.876304815306498 7.186596409982356
25	1000	5,10	6.875680471350503 7.187058190430975
26	2000	5,10	6.875678511939660 7.187029846340836
27	3000	5,10	6.873711908038665 7.188524343702240
28	4000	5,10	6.874458314655341 7.187974651468174
29	4300	5,10	6.875562301657048 7.187148868074652
30	4400	5,10	6.876379265468850 7.186566840788998
31	4500	5,10	6.877672400531351 7.185630263612991
32	4600	5,10	6.871336593835755 7.190427728450918
33	4700	5,10	6.876871670260329 7.186238025838104
34	4800	5,10	6.873849502336181 7.188557991463951

Table 4: Testing for the qualitative behaviour of the solution trajectories of real time mutualistic interacting systems for the initial data (6, 10) when the duration of growth ranges from 200 days to 4800 days

Example	Duration of growth in days	Initial data	Limiting Converging Values (times 100)
35	200	6,10	6.875107839773555 7.187470739399910
36	400	6,10	6.874845871578941 7.187689135880186
37	1000	6,10	6.874166391122195 7.188193967389592
38	2000	6,10	6.870655090736607 7.190781141555109
39	3000	6,10	6.875859070523469 7.186922853095608
40	4000	6,10	6.873589895699350 7.188626339447122
41	4300	6,10	6.874028142620936 7.188290623097523
42	4400	6,10	6.879648340864979 7.184131366663881
43	4500	6,10	6.870392949452147 7.190992417155578
44	4600	6,10	6.882428390968654 7.182223857058338
45	4700	6,10	6.869989482225576 7.191333026679556
46	4800	6,10	6.872629493483220 7.189476798671750

What do we learn from Table 5? When the initial data are 2 and 10 and when the length of the growing season is 4300 days, the co-existence steady-state solution (687.5, 718.76) can be considered to be approximately stabilized. A similar conclusion can be drawn from the data displayed in Table 6.

Table 5: Testing for the qualitative behaviour of the solution trajectories of real time mutualistic interacting systems for the initial data (2, 10) when the duration of growth ranges from 200 days to 4800 days

Example	Duration of growth in days	Initial data	Limiting Converging Values (times 100)

47	200	2,10	6.879961808538042 7.184022683267123
48	400	2,10	6.873922318556332 7.188454367559890
49	1000	2,10	6.875708356921938 7.187051919946174
50	2000	2,10	6.874297067158631 7.188091243309067
51	3000	2,10	6.875316370564589 7.187329440694295
52	4000	2,10	6.877104303151189 7.186027743104512
53	4300	2,10	6.875007519244421 7.187562528748780
54	4400	2,10	6.876183539743686 7.186680817102725
55	4500	2,10	6.873624262746179 7.188589071434775
56	4600	2,10	6.879612239631708 7.184162660999132
57	4700	2,10	6.870625664494043 7.190805621622640
58	4800	2,10	6.874508781219731 7.187932733793431

Table 6: Testing for the qualitative behaviour of the solution trajectories of real time mutualistic interacting systems for the initial data (1.5, 10) when the duration of growth ranges from 200 days to 4800 days

Example	Duration of growth in days	Initial data	Limiting Converging Values (times 100)
59	200	1.5,10	6.880212830969085 7.183873904218220
60	400	1.5,10	6.876499516358990 7.186505746368376
61	1000	1.5,10	6.869938358315727 7.191357348084612
62	2000	1.5,10	6.876055276351687 7.186782380072723
63	3000	1.5,10	6.874253487384720 7.188123544624772
64	4000	1.5,10	6.876701590563863 7.186303651351493
65	4300	1.5,10	6.873425548879213 7.188748350025879
66	4400	1.5,10	6.874541309196405 7.187912108625089
67	4500	1.5,10	6.876352418122663 7.186554481583271
68	4600	1.5,10	6.873903195631431 7.188379013763884
69	4700	1.5,10	6.878710939018828 7.184834594313262
70	4800	1.5,10	6.872197741687824 7.189627251585599

It is interesting to observe that the expected co-existence steady-state solution (687.5, 718.75) can be fully stabilized when the length of the growing season is 4000 days and the initial data are 0.5 and 10.

Table 7: Testing for the qualitative behaviour of the solution trajectories of real time mutualistic interacting systems for the initial data (0.5, 10) when the duration of growth ranges from 200 days to 4800 days

Example	Duration of growth in days	Initial data	Limiting Converging Values (times 100)

71	200	0.5,10	6.873574710734848	7.188628218951975
72	400	0.5,10	6.875685034026667	7.187053702380913
73	1000	0.5,10	6.874455167756764	7.187975439510715
74	2000	0.5,10	6.875311510028841	7.187355635037476
75	3000	0.5,10	6.880086577650523	7.183937548698344
76	4000	0.5,10	**6.875122296232479**	**7.187538081664957**
77	4300	0.5,10	6.871921561291601	7.189905539474710
78	4400	0.5,10	6.875895266235650	7.186904836487655
79	4500	0.5,10	6.874523751069512	7.187952036230968
80	4600	0.5,10	6.874196203770364	7.188193806447821
81	4700	0.5,10	6.875992869836876	7.186843023601025
82	4800	0.5,10	6.875946396173004	7.186880961726419

Conclusion

In this present study that concerns the mutualistic interaction between two species of yeast, it is hereby reported that the positive unique co-existence steady-state solution (687.5, 718.75) can be successfully stabilized when the length of the growing season is 4000 days while the initial data are 0.5 and 10.

References

[1] E. N. Ekaka-a, N. M. Nafo, N. E. S. Lale (2014), Numerical modelling of stability of a mutualistic interaction with changes in the intrinsic growth rate, Journal of the Nigerian Association of Mathematical Physics, Vol. 27, pp. 569-572.
[2] E. N. Ekaka-a (2009), Computational and mathematical modelling of plant species interaction in the harsh climate, PhD Thesis, The University of Liverpool and The University of Chester, United Kingdom.
[3] M. Kot (2001), Elements of Mathematical Ecology, Cambridge University Press.
[4] E.C. Pielou (1977), Mathematical Ecology, 2nd ed., Wiley, New York.
[5] J.D. Murray (1993), Mathematical Biology, 2nd ed., Springer, Berlin.
[6] E. Renshaw (1991), Modelling Biological Populations in Space and Time, Cambridge University Press.
[7] C. Damgaard (2004), Evolutionary Ecology of Plant-Plant Interactions: An Empirical Modelling Approach, Aarhus University Press, 2004.