Harvard Research and Publications International

June 30, 2019 Vol. 10 No. 7 ISSN: 0081 – 3998

JOURNAL OF AFRICAN SUSTAINABLE DEVELOPMENT

IBINAKU'S THEOREMS OF RIGHT TRIANGLES

IBINABO M. TAMUNO-OMIE & AKURO SEMINIWORIMA

Department of Mathematics, Faculty of Science, Rivers State University, Rivers State, Nigeria.

ABSTRACT

In this work, we present proofs and application of a derivative (Ibinaku's Theorems of Right Triangles) of Pythagoras' theorem. These theorems are to solve rightangled triangle and relevant problems in Euclidean Geometry.

Keywords: *Pythagoras, Pythagoras' theorem, Pythagorean triples, Ibinaku's theorem, Heron's formula*

INTRODUCTION

In the past years (B.C), **Pythagoras' theorem** is subject to lots of studies and contributions. The main approaches are Pythagorean triples on set of numbers **[Plimpton 322]** which today seems to be old fashioned due to the difficulty of applying sets of numbers that satisfy the equations. The values of n for which there are integral solutions to the equation, $x^n + y^n = z^n$, which has proved their efficiency for $n = 2$, in that the integral solution exists. **Pierre de Fermat (1666)** conjectured that there were no solutions for a value of $n>2$, without a proof though, he wrote in the margin of a textbook that the relationship wasn't possible, in the stead, but he did not have enough room in the page to pen it down. This became **Fermat's Last Theorem**, which appeared apparently simple, but it wasn't until the intervention of **Andrew Wiles** of **Princeton University**, who finally proved the theorem. **Andrew's (1992)** standard notation **Peter et-l (2011)** which explained the first and second version uses archaic language but both are Pythagoras' theorem.

Pythagoras's theorem is used in determining the distance between two points in both two and three dimensional space. This theorem can be generalized as cosine rule and it was used to establish **Heron's formula** for the area of a triangle, **MC – Ty (2009)**. Without the Pythagorean Theorem, none of the following is possible: radio, cell phone, television, internet, flight, pistons, and cyclic motion of all sorts, surveying and associated infrastructure, development and interstellar measurement sparks **(2008)**.

However, in this work, we propose the methodology of Ibinaku's Theorems of right triangles: The first theorem was obtained by the application of the 'tangent' trigonometric ratio. Second, the Quadratic formula was applied to determine the length of the Opposite and Adjacent sides. Third, necessary and sufficient conditions and proofs were pronounced. Furthermore, some practical examples were solved in this paper.

The outline of this paper is as follows: In the next section, the theorems were stated and proved. A formula was also derived to determine the length of the Opposite and Adjacent sides. Illustrative examples dealing with right triangles were attached to show the main methodology of Ibinaku's Theorems.

Having laid this foundation, let us quickly look into **Ibinaku's Theorems**.

Ibinaku's 1st Theorem states that:

In a given right triangle, the square of the hypotenuse equals the product of the sum of the tangents of the two complementary angles and the product of the opposite and adjacent sides.

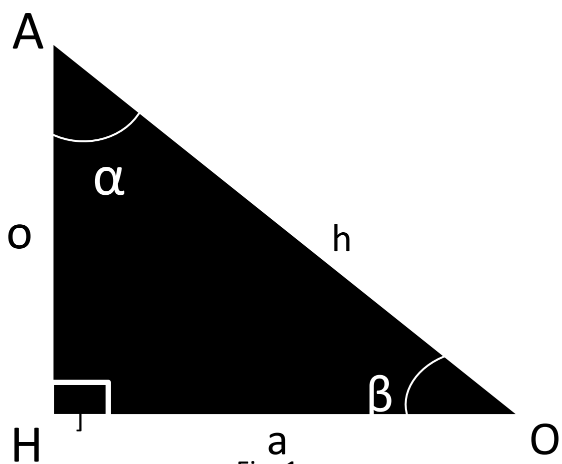

Fig. 1

2. Basic notation, preliminaries and definitions

Given: A right triangle AOH with OÂH = α and AÔH = β.

To prove: $h^2 = (\tan\alpha + \tan\beta) \times (oa)$

Proof: from Fig. 1

$$/AO/ = h = hypotenuse$$
$$/AH/ = o = opposite$$
$$/OH/ = a = adjacent$$
$$h^2 = a^2 + o^2 \ldots\ldots\ldots\ldots\ldots(1)$$
(Pythagoras' theorem)
$$\tan\alpha = {}^{a}/_{o}, \tan\beta = {}^{o}/_{a}$$
$$o^2 + a^2 = ({}^{a}/_{o} + {}^{o}/_{a}) \times (oa)$$
$$But\ (\tan\alpha + \tan\beta) = ({}^{a}/_{o} + {}^{o}/_{a})$$
$$Thus, o^2 + a^2 = (\tan\alpha + \tan\beta) \times (ao) \ldots\ldots (2)$$

By substituting (2) in (1), we have

$$\boldsymbol{h^2 = (\tan\alpha + \tan\beta) \times (ao)} \ldots\ldots\ldots\ldots \textbf{(3)}$$

Ibinaku's 2nd Theorem states that:

The square of the hypotenuse equals the quotient of the product of the square of the sum of the opposite and adjacent and the sum of the tangents of the two complementary angles, and the sum of 2 and the sum of the tangent of the two complementary angles.

With reference to Fig. 1 **Given**: A right triangle AOH with OÂH = α and AÔH = β.

To prove: $h^2 = \frac{(o+a)^2(\tan\alpha + \tan\beta)}{(\tan\alpha + \tan\beta) + 2}$

Proof: from Fig. 1

$$/AO/ = h = hypotenuse$$
$$/AH/ = o = opposite$$
$$/OH/ = a = adjacent$$

$h^2 = a^2 + o^2$ (Pythagoras' theorem)

$a^2 + o^2 = (a + o)^2 - 2(ao)$.................. (4)

By substituting (4) in (1), we have

$h^2 = (a + o)^2 - 2(ao)$.................. (5)

From (3),

$(ao) = \frac{h^2}{(\tan\alpha + \tan\beta)}$................................. (6)

Substitute (6) in (5)

$$h^2 = (a + o)^2 - 2[\frac{h^2}{(\tan\alpha + \tan\beta)}]$$

$$h^2 = (a + o)^2 - \frac{2h^2}{(\tan\alpha + \tan\beta)}$$

$$h^2 = \frac{(a + o)^2(\tan\alpha + \tan\beta) - 2h^2}{(\tan\alpha + \tan\beta)}$$

$$h^2 (\tan\alpha + \tan\beta) = (a + o)^2 (\tan\alpha + \tan\beta) - 2h^2$$
$$h^2 (\tan\alpha + \tan\beta) + 2h^2 = (a + o)^2 (\tan\alpha + \tan\beta)$$
$$h^2 [(\tan\alpha + \tan\beta) + 2] = (a + o)^2 (\tan\alpha + \tan\beta)$$

Thus,

$$\boldsymbol{h^2 = \frac{(a + o)^2 (\tan\alpha + \tan\beta)}{(\tan\alpha + \tan\beta) + 2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

<u>Finding the Length of the Opposite (o) and Adjacent(a) Sides</u> From (3)

$h^2 = (\tan\alpha + \tan\beta) \times (ao)$........................... (8)

From (7)

$$h^2 = \frac{(a + o)^2 (\tan\alpha + \tan\beta)}{(\tan\alpha + \tan\beta) + 2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$Put\ x = (ao)$……………………………………………… (10)

From (8)

$$(ao) = \frac{h^2}{(tan\ \alpha + tan\ \beta)}$$

That is,

$$x = (ao) = \frac{h^2}{(tan\ \alpha + tan\ \beta)}$$ ………………………..……(11)

$Put\ y = (a + o)$……………………………………………(12)

From (9)

$$y^2 = (a + o)^2 = \frac{h \quad 2 + \quad tan\ \alpha + tan\ \beta}{(tan\ \alpha + tan\ \beta)}$$

$$y = (a + o) = h\sqrt{\frac{2+(tan\alpha+tan\beta)}{(tan\alpha+tan\beta)}}$$ ……………………………… (13)

^{2} [( )]

From (10)

$$o = {}^{x}/_{a}$$ ……………………………………… (14)

From (12)

$$o = y - a$$……………………………………… (15)

Equating (14) and (15)

$${}^{x}/_{a} = y - a$$

$$x = ya - a^2$$

$$a^2 - ya + x = 0$$ ……………………………. (16)

Using the Quadratic formula with a as the variable and x, y as the constants then:

$$a = \frac{y \pm \sqrt{y^2 - 4x}}{2}$$ ……………………………... (17)

Substituting the initials for x & y

$$a = \frac{h\sqrt{\frac{[2+(tan\alpha+tan\beta)]}{(tan\alpha+tan\beta)}} \pm \sqrt{\frac{h^2[2+(tan\alpha+tan\beta)}{(tan\alpha+tan\beta)} - \frac{4h^2}{(tan\alpha+tan\beta)}}}{2}$$

$$= \frac{h\sqrt{\frac{[2+(tan\alpha+tan\beta)]}{(tan\alpha+tan\beta)}} \pm \sqrt{\frac{h^2[2+(tan\alpha+tan\beta)]-4h^2}{(tan\alpha+tan\beta)}}}{2}$$

]

$$= \frac{h\sqrt{\frac{[2+(tan\alpha+tan\beta)]}{(tan\alpha+tan\beta)}} \pm \sqrt{\frac{h^2[(tan\alpha+tan\beta)-2}{(tan\alpha+tan\beta)}}}{2}$$

$$a = \frac{h(\sqrt{2+(tan\alpha+tan\beta)} \pm \sqrt{(tan\alpha+tan\beta)-2})}{2\sqrt{(tan\alpha+tan\beta)}} \ldots \ldots$$ .... (18)

EXAMPLES

Example 1: Find the value of h.

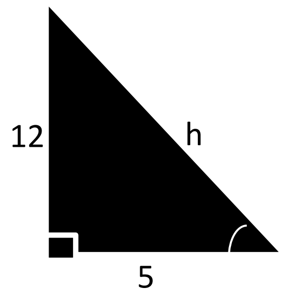

Fig 2

Using Pythagoras Theorem: $h^2 = o^2 + a^2$,

$o = 12cm$,

$a = 5cm$

$h^2 = 5^2 + 12^2$

$= 25 + 144$

$h^2 = 169 \rightarrow h = \sqrt{169}cm^2$

$h = 13cm$

<u>**Application of Ibinaku's Theorems**</u>

N/B: The following are paraphrased/rephrased questions of **Example 1**.

Example 2: Find the length of the hypotenuse of a right triangle given that the product of the opposite and adjacent sides is 60cm^2 and the sum of the tangents of the two complementary angles is 2.8167^0

Solution

$(ao) = 60cm^2$

$tan\ \alpha = 0.4167$

$tan\ \beta = 2.4$

By Ibinaku's 1st Theorem,

$h^2 = (tan\ \alpha + tan\ \beta)\ (oa)$

By substitution, we have that

$h^2 = 2.8167 \times 60cm2$

$h^2 = 169.002cm^2 \Rightarrow h = \sqrt{169}$

$h = 13cm$

$.002cm^2$

Example 3: Find the length of the hypotenuse of a right triangle given that the sum of the opposite and adjacent sides is 17cm and the sum of the tangents of the two complementary angles is 2.8167^0

Solution

$$(\quad) \qquad (\qquad \beta)$$
$$o + a = 17cm; \quad \tan\alpha + \tan \quad = 2.81670$$

By Ibinaku's 2nd Theorem

$$h^2 = \frac{(o + a)^2 (\tan\alpha + \tan\beta)}{2 + (\tan\alpha + \tan\beta)}$$

By substitution, we have that

$$h^2 = \frac{(17)^2 \times 2.8167}{2 + (2.8167)}$$

$$= \frac{814.026}{4.8167} cm^2 = 169.000cm^2$$

$$h = \sqrt{169}cm^2 = 13cm$$

Example 4: Given that the hypotenuse of a right triangle is 13cm, and the sum of tangents of the two complementary angles is 2.8167^0, find the length of the adjacent and opposite sides.

Solution

$$h = 13cm$$
$$(\tan\alpha + \tan\beta) = 2.81670$$

Recall,

$$\boldsymbol{a} = \frac{\boldsymbol{h}(\sqrt{\boldsymbol{2+(tan\alpha+tan\beta)}} \pm \sqrt{\boldsymbol{(tan\alpha+tan\beta)-2}})}{\boldsymbol{2}\sqrt{\boldsymbol{(tan\alpha+tan\beta)}}}$$

By substitution, we have:

$$a = \frac{13(\sqrt{\mathbf{2+(2.8167)}} \pm \sqrt{\mathbf{(2.8167)-2}})}{2\sqrt{2.8167}}$$

$$= \frac{13}{2}\left(\frac{\sqrt{4.8167} \pm \sqrt{0.8167}}{\sqrt{2.8167}}\right) = \frac{13}{2}\left(\frac{2.1945 \pm 0.9037}{1.6783}\right)$$

$$a = \frac{13}{2}\left(\frac{2.194 + 0.9037}{1.6783}\right) \quad \text{or} \quad \frac{13}{2}\left(\frac{2.194 - 0.9037}{1.6783}\right)$$

$$= \frac{13}{2}\left(\frac{3.0984}{1.6783}\right) \quad \text{or} \quad \frac{13}{2}\left(\frac{1.2908}{1.6783}\right)$$

$= 6.5\,(1.8462)$ or $6.5\,(0.7691)$

$= 12cm$ or $5cm$

$If\ a = 12cm, then\ o = 5cm. And\ if\ a = 5cm, then\ o = 12cm$

$$a = 12cm\ and\ o = 5cm$$

Note: *The second value of the adjacent side, a, is also the value of the opposite side, o.*

Conclusion

These theorems are subject to reprove. One unique feature in them is the ability to find the value of the opposite alongside the adjacent using the Quadratic formula, provided the value of the hypotenuse and the sum of the tangents of the two complementary angles are given.

References

Peter Brown, Michael Evans, David Hunt, Jamine McIntosh, Bill Pender, Jacqui Ramagge, *The Improving Mathematics Education in Schools (TIMES) Project*, AMSI (Australian Mathematical Sciences Institute), *June 2011*.

Cheng-Yao Lin, *Pythagorean Theorem Proofs*, ON-Math-2006-2007, Volume 5, Number 1.

Charles Frohman, *The Full Pythagorean Theorem*, *January 1, 2010*.

Manjil P. Saikai, *The Pythagoras' Theorem*, arxiv: 1310.0986v2 [math.HO], *22 Oct., 2013*.

John C. Sparks, *The Pythagorean Theorem*, Crown Jewel of Mathematics, Author House: xxxxx, 1663 Liberty Drive, Suite 200 Bloomington, Indiana 47708: *2008*.

Kamel Al-Khaled & Ameen Alawneh, *Pythagorean Theorem: Proofs and Applications*, Department of Mathematics and Statistics, Jordan University of Science and Technology, IRBID 22110, JORDAN.