ANALYSIS ON DEFLECTION OF STRUCTURAL ELEMENTS [A CASE STUDY OF BEAMS]

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Abstract: - Structural Reinforced Concrete Beams usually deflects from its original position. The amount by which Beams deflects depends upon its cross sectional area. And the bending moment. In modern design offices the two major Criteria for the deflection of Beams are the strength and the stiffness. The designed Beams are usually expected to be strong enough to resist the bending moment and shear force. Beams are also expected to be stiff enough to resist the deflection or not to deflect more than the permissible limit under the action of loadings. This paper Examines the deflection of Reinforce Beans, factors affecting deflection of Beams, different methods for determination of deflection,, design application methods, Effects of deflection, and control of deflection. The paper concluded with recommendation.

Keywords: *Deflection, Structural Elements, Reinforced concrete, Beams.*

Introduction: - structures and their Deflection
Structures, like all other physical bodies deform and change their shapes when they are subjected to forces. The other common causes of deformation in structures support settlements. If the deformation in a structure disappear with Time and the structure finally regains its original shape when the action causing the deformations are remove, such deformations are termed ELASTIC Deformation .the permanent deformation of structures are referred to as INELASTIC OR PLASTIC DEFORMATION. This paper focus on Linear Elastic deformation of Beams. It is known that such deformation vary linearly with applied loads, with understanding of the lords acting on the structures are double, and its deformation will also be double. The excessive deformations of most structures are undesirable, and such may impair the structure's ability to serve its intended purpose. Structures are usually designed so that their deflections under normal service conditions will not exceed the allowable values specified in the designed codes.

Deflection of Reinforced Concrete Beams

Deflection Of beams is defined as the distance between its original position before and after loading of Beams. It is usually denoted by letter noted by letter y. the deflection at any end of beams is usually taking as zero (o). et al (2008) defined deflection as the vertical downward displacement of the point from its original horizontal position and is measure to the neutral line of the Deformed Beams from 6the original neutral line [figure (i) and (ii)]

Slope of Beams

The slope at any section in a deflected beam is defined as the angle in radiance, which the target at the original axis of the beam. The slope at any point of a deflected beam is very small. Slope is usually denoted by θ^c figure (III)

(i) From figure (IV) above, slope is denoted by θ^c, which is measured in radian

(ii) Radian is the angle subtended at the centre by a length of radius

Factor Affecting Deflection of Beams

i. **Loads: -** When a Bean is Simply Supported, the deflection Increase as the leads Example: - Assuming that a point wad win is applied at the near Span of a simply supported being produces the deflection of ymm if the load is increased to 2WKN or 3WLAU, The length being constant, the deflection due to these loads will be 2ymm and 3ymm respectively. Therefore, deflection increases as the loads on the Beans

Loads Deflections

ii. **Spans of Beans: -**It has been proved experimentally and Aftermath call that the deflection of Beans is proportional to the cube of the span. It the span of any Bean is increased to 3L, its deflection will be 27 Time that of the Beans. It other words, the deflection of Beans is proportion al to the cube of the span'

iii. **Sizes and Shape of Beans:-**The greater the si3e and shape of Beans the Leased the deflection. The deflection of any Beans with respect to si3e and shape is inversely proportional to the moment of Inertia i.e $I = BD^3/12$

Where I = moment of Inertia, B = Breadth of the Beam,
and D = Depth of the beam.

iv. **Stiffness of materials:-** With Beams having materials, the less will be the deflection. The measure of the stiffness of a material and its deflection is inversely proportional to the value of E.

I.e YEI = WL

Y = WL/EI

Where W = deflection increase as the Load

Where L = The deflection of Beams which is proportional to the cube of Beams Length.

Where E = Deflection which is inversely proportional to the modulus of Elasticity (E)

Where I = Deflection which is inversely proportional to the moment of Inertia of Beams $1^{m}e$

$I = BD^3/12$

Methods for determing deflection of Reinforced Concrete Beams

There are different methods for computing the deflection of Reinforced concrete Beams. The common ones are as follows:-

1. Slope deflection Equations.
2. Moment distribution method
3. Macaulay's method for slope and deflection
4. Williot mohr's Theorem Application of moment Area method.
5. Work – Energy method or virtual work method
6. Supper position method
7. Conjugate Beam method
8. Elastic Energy method
9. double integration method for slope and deflection

Typical Structural Analysis for Determining Deflection of Loaded Beam with the Method of Slope Deflection Equations.

The slope –deflection:- Equations method for analyzing structural beams and frames was developed by Professor George A. many of the University of Minnesota in 1915. The fundamental principle of this method is that it expresses the moment at the end of a member in terms of the following parameters:-

(i) Fixed end moments

(ii) The rotations of the target at each end of the elastic carve of the member.

Basic Slope Deflection Equations

(i) $M_{AB} = M^{F}_{AB} + 2EI \quad [2\Theta A + \Theta_B - \underline{3}^{\delta}]$

(ii) $M_{BA} = M^{F}_{BA} + 2EI \quad [2\Theta_A + - \underline{3}^{\delta}]$

The above two equations caned so be expressed in this form, based on either point loads uniformly distributed loads or even the combination of both:-

iii. $M_{AB} = M_{AB} + 2EK$ [20 A + 0 B-3R]

IV. $M_{AB} = M_{AB} + 2$ Ek {D+20B-3R}

Where M_{AB} Actual moment at End A

Where M_{AB} Actual moment at End B

Where Mf_{AB} fixed End moment at End A

Where $Mf_{BA =}$ fixed End moment at End B

Where k = stiffness factor of member AB = I/L,

But $I = Bd^3/12$

Where B = Breadth or width

Where D = depth of the beam material

Where k = displacement factor = A/L

Where O_A = rotation of End A

Where O_B = rotation of End B

Illustration (ii) completely Analyze the beam shown below with point and uniformly distributed load

$$M_{AB} = M_{FAB} + \frac{2EI}{L} [2\theta_A + O_B -- \frac{30L}{L}] \_\_\_\_\_\_\_ \text{(i)}$$

$$M_{BA} = M_{FBA} + \underline{2EI} [\theta_A + 2\theta_B - \underline{30}] \_\_\_\_\_\_\_\_\_\_ \text{(ii)}$$

L L

$M_{BC} = MF_{BC} + \frac{2EI}{L} [2\theta_B + \theta^C -- \frac{30}{L}]$ _________(iii)

$M._{CB} = MF_{CB} + \frac{2EI}{L} [O_B + 2\theta_C - \frac{30}{L}]$ _________(iv)

$M.F_{AB} = -- \frac{PL}{8} = \frac{36X4}{8} = -- \underline{18KN}$

$M.F_{AB} = + \frac{PL}{8} = \frac{36X4}{8} = -- \underline{18KN}$

$MF_{BC} = \frac{WL}{12} = \frac{26X4^2}{12} = -- 34.67KN$

$MF_{CB} = \frac{WL}{12} = \frac{26X4^2}{12} = + 34.67KN$

$MF_{AB} = M^F_{AB} + \frac{2EI}{L} [2\theta_A + \theta_B -- \frac{30}{L}]$

$M_{AB} = -- 18 + \frac{2EI}{4} [O + \theta B -- O],$

Where OA = O i.e fixed end, and where O^C = O, because there is NO sinking or settlement at fixed end, or the beam is rigidly clamped there. Therefore the slope at the end is zero.

$= 18 + \frac{2EI}{4} \theta B$

$= \frac{1}{2} EI \theta B - 18$ _______(1)

$M_{BA} = MF_{AB} + \frac{2EI}{L} [\theta A + 2\theta B - \frac{30^C}{L}]$

$=18+2EI [0 + 2\theta B -- 0]+2EI(0+2\theta B-0)$

Where θA=0 fixed end, 8=0 because there Is NO Sinking or settlement at fixed end, or the beam is rigidly clamped there. Therefore the slope at the end is zero.

$=18 + 2 \frac{EI\theta B}{4}$

$=18 + {}^1/_2 EI\theta B$

$= {}^1/_2 EI\theta B + 18$_______(2)

$MBC = M^F_{BC} + \frac{2EI}{L} \{2\theta B + \theta_C - \frac{3\delta}{L}\}$

$=34.67 + 4EI (2\theta B + 0-0) (\theta c=0)$

Fixed end, δ=0, because there is no sinking or settlement at fixed end, or the beam is rigidly clamped there. Therefore the slope at the end is zero

$=34.67 + \frac{4EI\theta B}{4}$

$=EI\theta B + 34.67$_____________(3)

$M_{CB}= M^F_{CB} + \frac{2EI}{L} (\theta_B + 2\theta_C - \frac{3\delta}{L})$

$=34.67 + \frac{2EI}{4} (\theta_B + 0-0)$ $(\theta_C = 0)$

There is No settlement.

$34.67 + \frac{2EI\theta B}{4}$

$= ^1/_2\ EI\theta_B + 34.67$ ____________(4)

Equilibrium Equation

£$M_B = 0$

$M_{BA} + M_{BC} = 0$

Addition of Equations (2) and (3) = 0

½ ETθA + 18=0

EIθ A + 36 _________(2)

EIθA + 34.67 _______ (3)

EIθA + 36 + EIθA -- 34.67

2EIθA =1.33

$EI\theta A = \frac{1.33}{2} = 0.67$

EIθA = 0.67

Substitute $EI\theta_B$ in the slope deflection Equation of Eqn (1)

$M_{AB} = ^1/_2 EI\theta_B$ --18

$M_{AB} = ^1/_2 (0.67)$ --18

M_{AB} = 0.335 --18

M_{AB} = -- 17.67kn

$M_{BA} = ^1/_2$ EIθB + 18

$M_{BA} = ^1/_2$ EIθB+18=0

$M_{BA}{}^1/_2$ 9(0.67)+18

M_{BA}=0.335 +18=18.335KN

$M_{BC} = EI\theta_B + 34.67$

M_{BC} = 0.335 + 34.67

M_{BC} = 35.005KN/M

$M_{CB} = ^1/_2$ EIθB +34.67

M_{CB} = (0.335) +34.67

M_{CB} = 35.005KN/M

Bending Moment Diagram

Typical structural Analysis for determining deflection of loaded beam with moment distribution_method

With the development in advanced Engineering and Technology the slope deflection method is not being encouraged in the design offices simply because of universal use of moment distribution method.

The moment distribution method was introduced by professor Handy cross in1930. This method has been widely used for the analysis of different types of indeterminate structures. The method assumed all members of the structure to be fixed in position and direction, and equally fixed end. The moments due to external loads are usually computed along with other necessary parameters needed from any loaded beam.

Illustration (1) A TYPICAL ROOF
Beam is selected from one of the residential building. Completely analysis the beam with method of moment distribution method.

Solution_

Stiffness factor:

$K_{AB} = \frac{3}{4} \times \frac{EI}{L} = \frac{3}{4} \times \frac{1}{5} =$ o.15

$K_{BC} = K_{CD} = \frac{EI}{L}$ 1/5 = 0.20

$K_{DE} = \frac{3}{4} \times \frac{EI}{L} = 1/4 = 0.188$

Distribution Factor

I. At Support A \$ E: D $F_{AB} = D.F_{ED} = 1.0$

(Simply Supported End)

II. At support B : $D.F_{BA} = \frac{K_{AB}}{K_{BA}+K_{BC}} = \frac{0.15}{0.15+0.2} =$ **0.429**

$D.F_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = \frac{0.20}{0.15+0.20} =$ **0.571**

III At Support C $D.F_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{0.20}{0.20+020} =$ **0.5**

$D.F_{CD} = \frac{K_{CD}}{K_{CD}+K_{CD}} = \frac{0.20}{0.20+0.20} = 0.5$

IV. At Support $D: F_{DC} = \frac{K_{CD}}{K_{DC}+K_{DE}} = \frac{0.20}{0.20+0.188} = 0.515$

$D.F_{DE} = \frac{K_{DE}}{K_{DC}+K_{DE}} = \frac{0.188}{0.20+0.188} = 0.485$

Fixed end Moment.

1. $FEM_{AB} = FEM_{ED} = 0$

2. $FEM_{BA} = \frac{W1^2}{8} = \frac{10X5^2}{8} = 31.25K^N m.$

3. $FEM_{BC} = -\frac{w1^2}{12} = -\frac{12x5^2}{12} = -25\ K^N m.$

$FEM_{CD} = +\frac{W1^2}{12} = +\frac{12X5^2}{12} = +25K^N m$

4. $FEM_{DC} = -\frac{w1^2}{12} = -\frac{8X5^2}{12} = -16.\ 67K^N m.$

$FEM_{DC} = -\frac{w1^2}{12} = +\frac{8X5^2}{12} = +16.67K^N m$

5. $FEM_{DE} = -\frac{w1^2}{8} = -\frac{12X4^2}{8} = -24K^N m.$

D.F	1.0	0.429	0.571 0.5	0.5 0.515	0.485	1.0
FEM	0	+31.	25 .0 +25.0	-16.67+16. 16	-240	0
Distr.mon	0	-2.68	3-57 -4.17-	417 +3.77	+3.56	0
Com	0	0.	-2.09 -1.79	+1.89 -2.0 9	0	0
Distr.mon	0	+0.90	+1.19-0.05	5-05 +1.08	+1.01	0
Com	0	0	-0.03+0.60	+0.54 -0.03	0	0
Distr.mon	0	+0.01	+0.02-0.57	-0.57 +0.02	+0.01	0
Com	0	-0.29	-0.29+0.01	+0.01 -0.02	0	0
Distr.mon	0	+0.12	+0.17-0.01	-0.01 +0.15	+0.14	0
E	0	+29.6	-29.6 +19.02	-19.03 +19.28	-19.28	0

Shear Force Diagram

Bean AB =>   =

$R_A = R_{BI} = {}^{WI}/_2 \; \frac{10X5}{2} = 25KN$

Bean $B_{C =>}$ $R_{B2} = R_{CI} = \frac{wl}{2} = \frac{12x5}{2} = 30KN$

Bean CD=> $R_{C2} = R_{DI} = \frac{WI}{2} = \frac{8X5}{2} = 20KN$

Bean D_E=> $R_{D2} = R_E \frac{WI}{2} = \frac{12X4}{2} = 24KN$

$R_B = R_{BI} + R_{B2} = 25 + 30 = 55kN.$
$R_C = R_{CI} + R_{C2} = 30 + 20 = 20KN.$
$R_D = R_{DI} + R_{D2} = 20 + 24 = 44KN.$

$S.F_{before}$ B = 25 - IV (5) + 55 -12 (5) = - 25
$S.F_{before}$ C = 25- IV (5) + 55 -12 (5) + 50 -8 (5) = - 20
$S.F_{before}$ D= 25 - IV (5) + 55 -12 (5) + 50 -8 (5) = -20
$S.F_{before}$ E=25 - IV (5) + 55 -12 (5) + 50 -8(5) + 44-12 (4) = - 24.

Bending Moment Diagram
Span AB $=M_{AB}^0$ =0.125 X 10 X 25 = 3125KN.m
Span BC $=M_{BC}$ =0.125 X 12 X 25 = 37.5KN.m
Span CD $=M_{CD}$ =0.125 X 8 X 25 = 28KN.m.
Span DE $=M_{DE}$ =0.125 X 12 X 16 = 24KN.m.

Span Moment $M^o = M^{o-}$ m+m^2 {m$ m^2 are moment at Support}

$M_{AB} = M^o_{AB} - \frac{M_A + M_B}{2} = 31.25 - \frac{0+29.6}{2} = 16.45$ KN.

$M_{BC} = M^o_{BC} - \frac{M_B + M_C}{2} = 37.5 - \frac{29.+19.02}{2} = 13.19$KN.m

$M_{CD} = M^o_{CD} - \frac{M_C + M_D}{2} = 25 - \frac{19.02+19.28}{2} = 5.85$KN.m

$M_{DE}\ M^o_{DE} - \frac{M_D + M_E}{} = 24 - \frac{19.28+0}{} = 14.34$KN.m.

Bending Moment Diagram

Effects of Deflection

The following are the effects of deflection on any Structure

I. Excessive deflections and deformations can impair the appearance and efficiency of any structures(eog) pain forced concrete Beam) and also cause discomfort or alarm to the occupants'

II. Excessive deflections can cause cracking and possible separations of plaster finishes, crushing of partition walls or cracking of gla3ing unity.

Control of Deflection

The maximum deflections which are usually premised by design codes (e.g Bs silo) under normal working loads are usually in terms of spans. or height of Beams Consternation of experiences has indicated that deflections are likely to sati's factory if certain limiting span to effective depth ratios are not exceeded.

The vertical deflection Limits of Beams are generally assumed to be in control or satisfied provided that the spans to effective depth rations are not greater than the values obtained as follows during the design:

(a) For span up to 10m:-

(I) Cantilever = 7

(II) Simply Supported = 20

(III) Continuous = 26

Recommendation

The occurrence of defection on structural pain forced. Concrete Beams will be Inequitable if necessary chider are not put into consider ration during the design supervision and contention of strictest Elements. It of structures that designers of structure bear in such as the loads, mind those factors, spans si3es and shapes of the Beams, and the proper stiffness' of the internals when clarifying out their design proper would help to reduce such factors deflections.

Conclusion

The general requirements in the design of structural Elements, especially Beams is that neither the efficiency nor the appearance of structures is harmed by the deflection that will occur during the useful life of structural Elements Deflection should thus be considered at various shaper The limitation necessary to satisfy the require wants will vary considerably according to the nature of the structural Element in mind and its loadings but in general term for reinforced concrete the following should be considered as reasonable guides:-

i. The final deflection of a Beam, slab or can tiller should not exceed $\frac{span}{250}$

ii. The part of the deflection which takes place. After the application of finishes or the introduction of partitions should not exceed the $\frac{Span}{250}$ to colloid damages to fixtures and fittings

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